Exploiting duality using metamaterials

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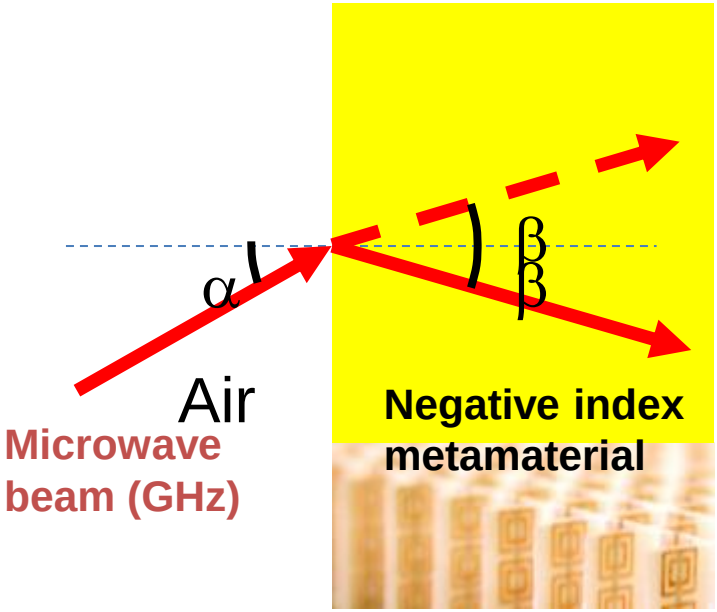
Metamaterials with magnetic response

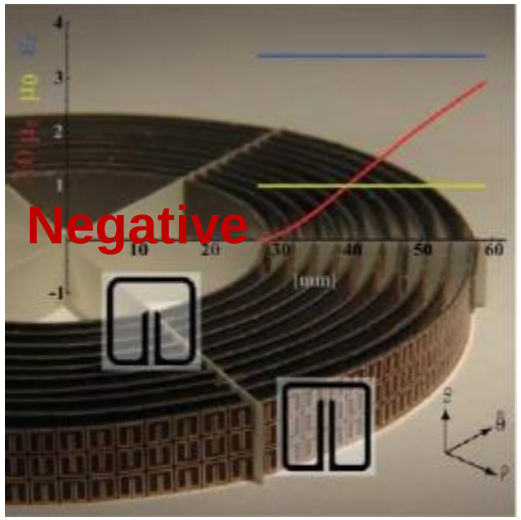
- Interesting physics by pairing up electric and magnetic responses

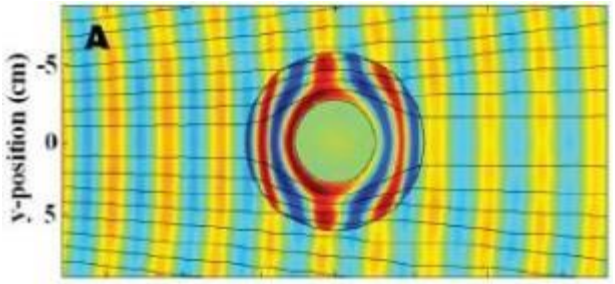


Shelby, Smith & Shultz, Science 2001

Negative refractive index

Cloak





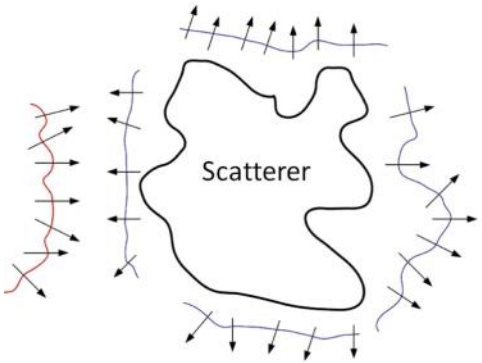
Schurig, Smith & Pendry, Science 2006

Movitation

- Electromagnetic duality is symmetry of microscopic Maxwell's equation but broken when materials are present

$$\mathbf{E} \rightarrow \mathbf{E}_\theta = \mathbf{E}\cos\theta - \mathbf{H}\sin\theta,$$
$$\mathbf{H} \rightarrow \mathbf{H}_\theta = \mathbf{E}\sin\theta + \mathbf{H}\cos\theta.$$

$$\mathbf{E} \rightarrow \mathbf{H} \text{ and } \mathbf{H} \rightarrow -\mathbf{E}, \text{ for } \theta = -\pi/2$$



Decoupled into TE/TM rather than LCP/RCP
Unless $\epsilon = \mu$
Possible if we have metamaterials

Ivan Fernandez-Corbaton et.al., PRL (2013)

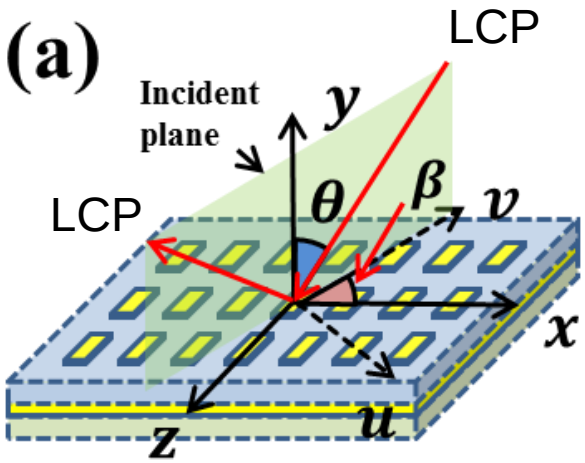
Outline

- **Investigate phenomena driven by different kinds of dual operations between electric and magnetic fields**
- **Enabled by metamaterials by putting electric & magnetic fields in equal footing**

- Restoring duality with metamaterials
  - Helicity-preserving mirror
- Redefining parity operator using duality
  - Demonstrating Parity-Time-phase transition and unidirectional reflection at exceptional point
- Gauging dual operation
  - Establishing pseudo magnetic field with metamaterials

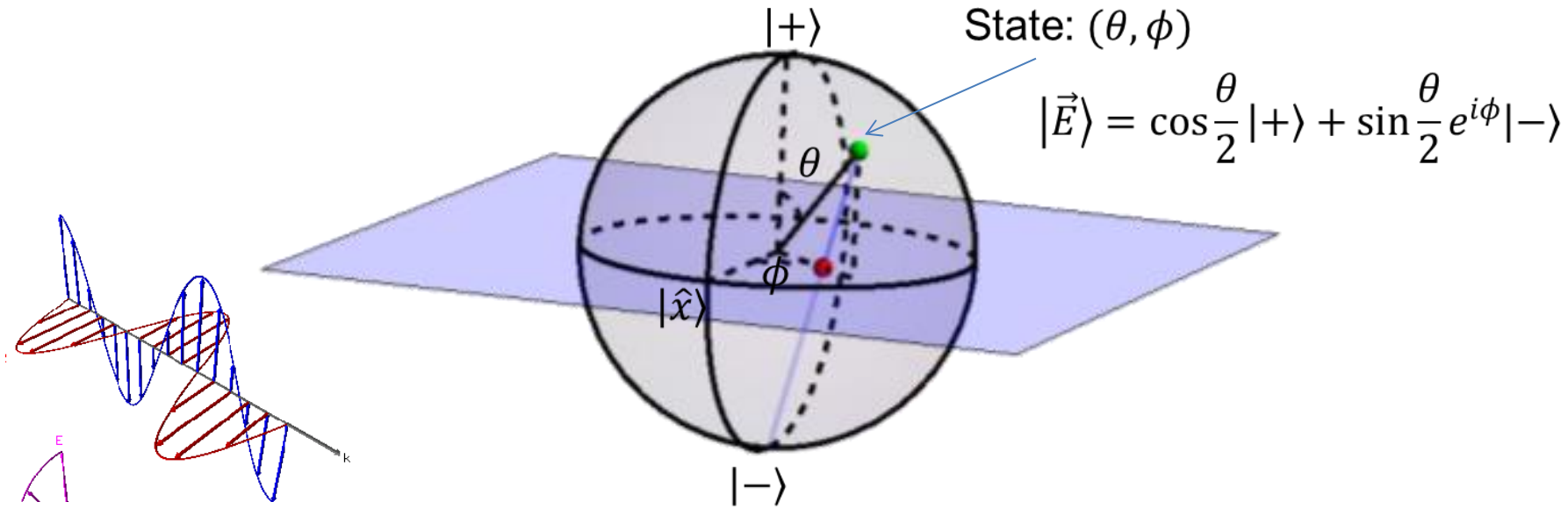
Restoring duality with metamaterials

- Helicity/spin angular momentum of photon reversed for a normal mirror at any angle
- Target: reflecting metasurface preserving helicity at all angles
  - An example of metasurface to have all-angle polarization control



Spin/polarization state

- A polarization lives on Poincare sphere
- North pole: Left handed circular polarization
- South pole: Right handed circular polarization



LCP: $|+\rangle = (\hat{\mathbf{x}} + i\hat{\mathbf{y}})/\sqrt{2}$

RCP: $|-\rangle = (\hat{\mathbf{x}} - i\hat{\mathbf{y}})/\sqrt{2}$

Helicity-preserving mirror

Criteria at normal incidence

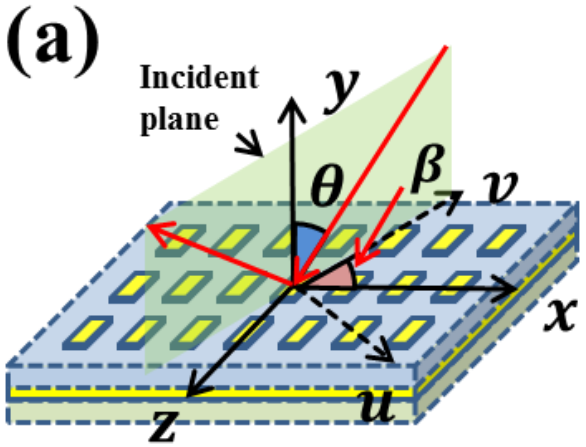
Normal reflection phase: Φ_z, Φ_x
with $\Phi_x - \phi_z = \pi$

Sub-λ structures
& thickness

Effective boundary condition

$$E_z + iH_x \cot\frac{\Phi_z}{2} = 0,$$

$$E_x - iH_z \cot\frac{\Phi_x}{2} = 0,$$

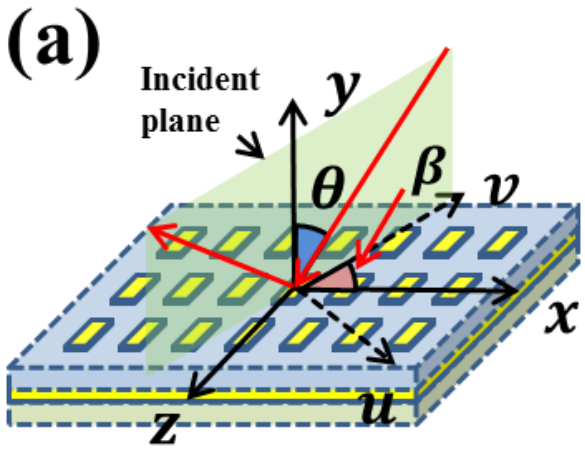


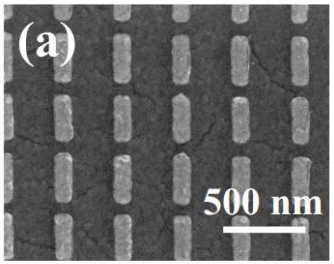
Decoupled into circular polarizations

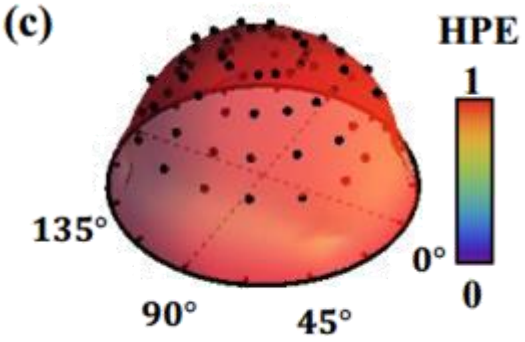
$$(\mathbf{E} \pm i\mathbf{H}) \cdot (\hat{u}\sin(\Phi_z/2 \pm \beta) \pm \hat{v}\cos(\Phi_z/2 \pm \beta)) = 0$$

Helicity-preserving mirror

- Normal mirror flips the spin of light at any angles
- Spin-preserving mirror has boundary condition:







$$r^{\pm}(\theta,\beta)=\exp(2i\tan^{-1}(\frac{\tan(\Phi_z/2\pm\beta)}{\cos\theta}))$$

Normal incidence

$$r^{\pm}=\exp[i(\Phi_z\pm 2\beta)]$$

Geometric phase

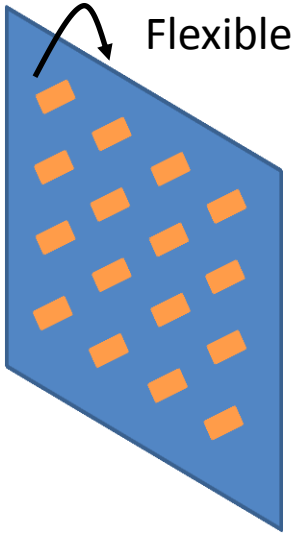
Gold nano-rods on gold mirror with MgF2 spacer

Theory: IEEE Trans. Antennas & Propag. 62, 6274 (2014)
Experiment: Adv. Opt. Materials (accepted)

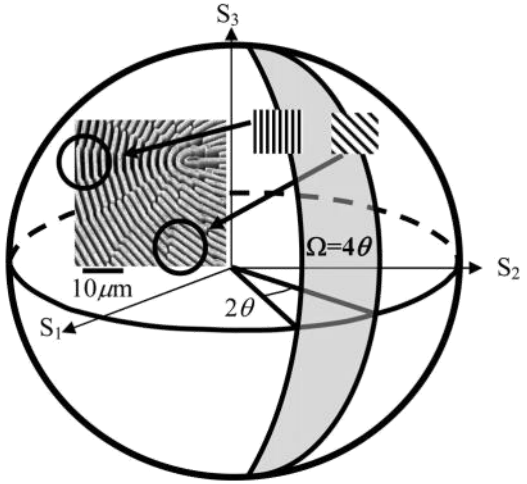
Geometric phase in polarization space

Flexible phase by rotating the structures

Incident wave
with LCP



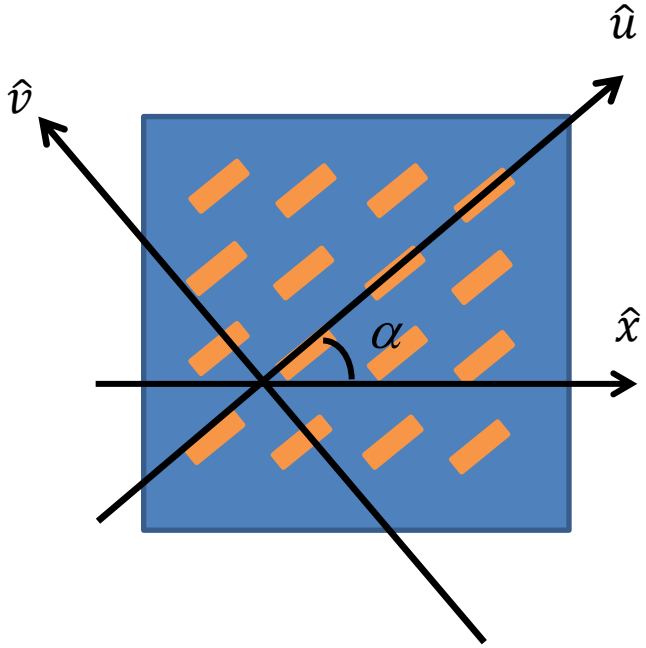
transmitted wave
of cross-polarization RCP
with additional transmitted phase Φ_{Berry}



A. Niv et. al. Opt. Lett. (2007)

$$\Phi_{Berry} = \frac{1}{2}\Omega = 2\alpha$$

Enclosed area



Cross-polar. Conversion:
$|\pm\rangle \rightarrow e^{\pm i2\alpha}|\mp\rangle$

Geometric phase in polariztion space

- Opposite geometric phases for the two spins

Material:

$$E_{tu} = t_u E_u, \qquad E_{tv} = t_v E_v$$

$$\hat{x} \pm \hat{y} = \exp(i\alpha)\,(\hat{u} \pm i\hat{v})$$

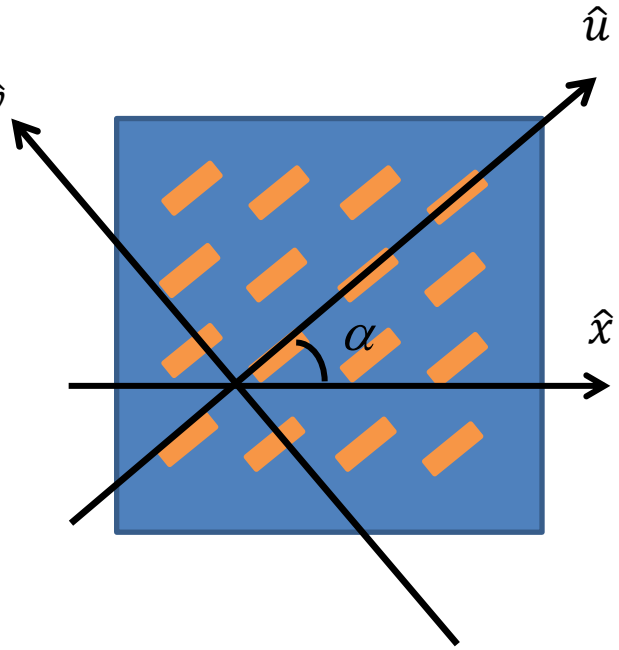
$$\begin{pmatrix} E_+ \\ E_- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} E_u \\ E_v \end{pmatrix}$$

$$\begin{pmatrix} E_{t+} \\ E_{t-} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} t_u & 0 \\ 0 & t_v \end{pmatrix} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \begin{pmatrix} E_+ \\ E_- \end{pmatrix}$$

$$= \begin{pmatrix} (t_u + t_v)/2 & \exp(-i2\alpha)(t_u - t_v)/2 \\ \exp(+i2\alpha)(t_u - t_v)/2 & (t_u + t_v)/2 \end{pmatrix} \begin{pmatrix} E_+ \\ E_- \end{pmatrix}$$

$$\Phi_{Berry} = \frac{1}{2}\Omega = 2\alpha$$

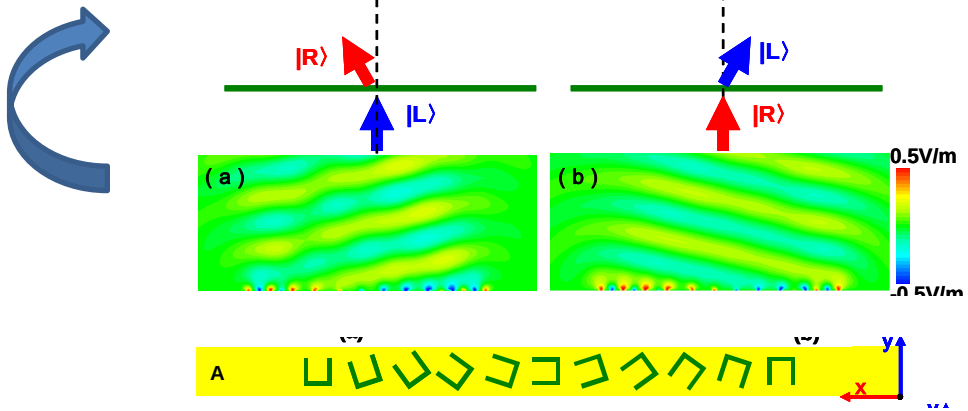


App: Spin-induced orbital manipulation

- Spin-dependent refraction

$$k_0(n_t \sin\theta_t - n_i \sin\theta_i) = \frac{\partial \Phi_{Berry}}{\partial_x}$$

Spin coupled to orbital motion



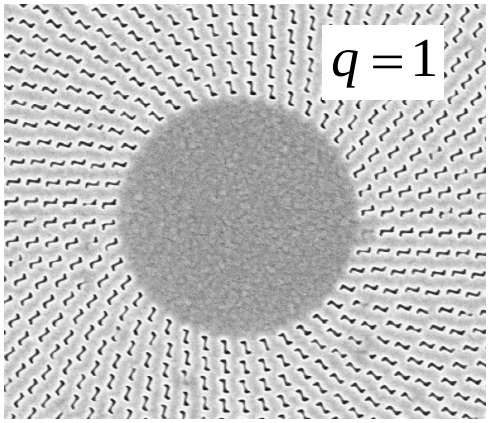
$$\Phi_{Berry} = \pm 2\alpha$$

Spin-dependent refraction

Opt. Exp. 20, 15882 (2012)

App: Spin-induced orbital manipulation

- Inducing orbital angular momentum for angular variation

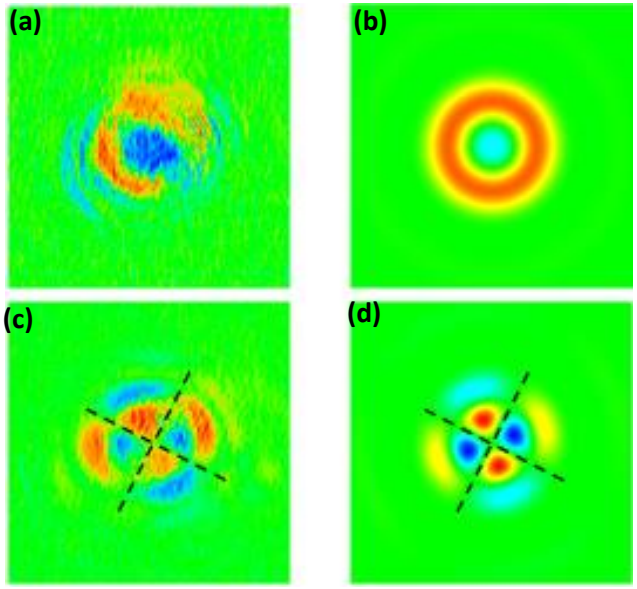


$$|l=0,+\rangle \rightarrow |l=2,-\rangle$$
$$|l=0,-\rangle \rightarrow |l=-2,+\rangle$$

Experiment Theory

RCP as incident

Linear y-polarization as incident

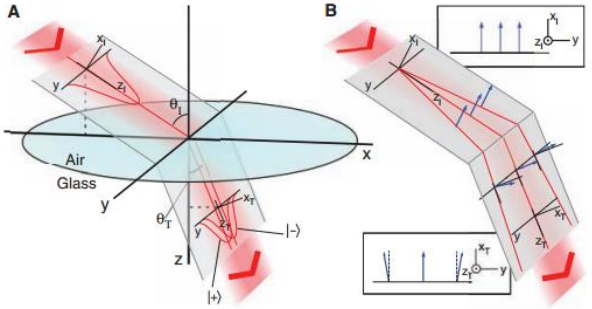


Stokes S_3 parameter plotted
Red: LCP
Blue: RCP
Additional rotation of orbitals by around 27°

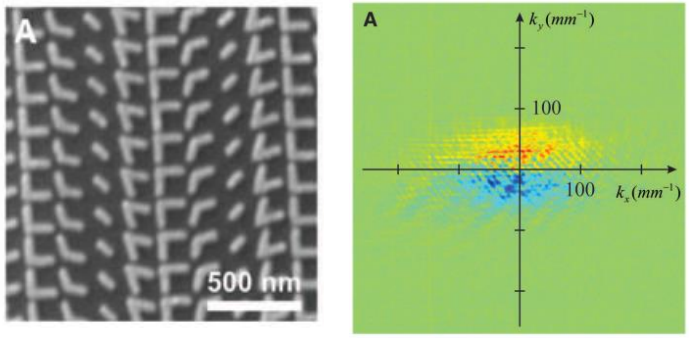
Nano Lett. 13, 4148 (2013)

Optical Spin-Hall Effect (OSHE)

- Refraction



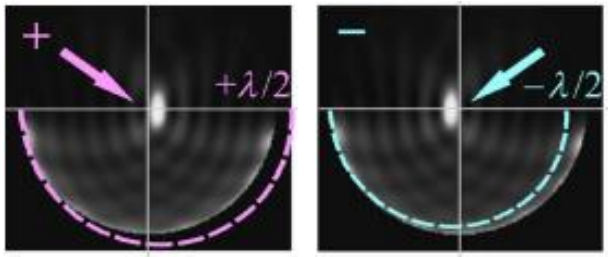
Air-glass interface
Hoston, Science (2008).



Metasurface
X. Yin, Z. Ye, J. Rho, Y. Wang & X. Zhang, Science (2008).

Spin can affect orbital motion thru opposite geometric phases: spin Hall effect of light

- Surface-plasmon-polariton (plasmonic spin-Hall effect)

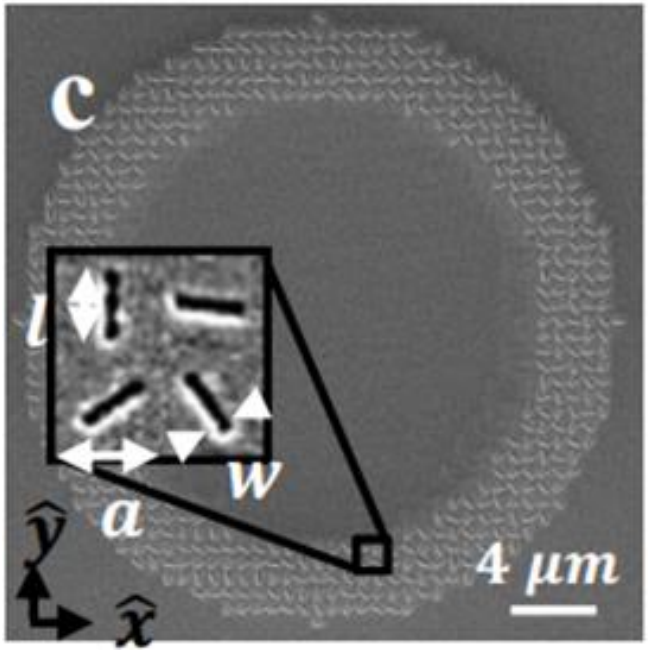


K. Y. Bliokh et.al., PRL (2008).

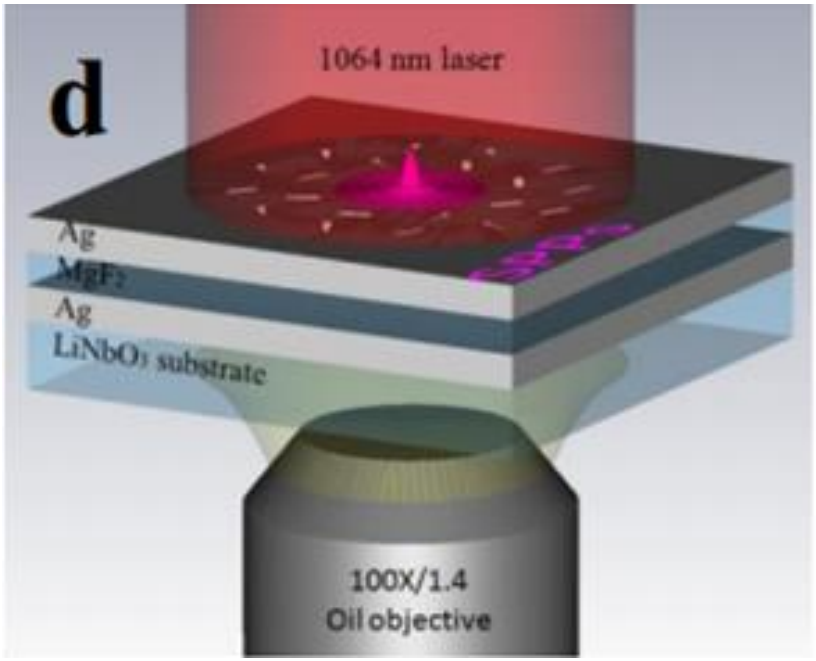
The splitting is tiny (comparable to wavelength)

App: Holographic interface for SPP

- A generic manipulation of OSHE
- Arbitrary specification of target profiles for the two spins possible



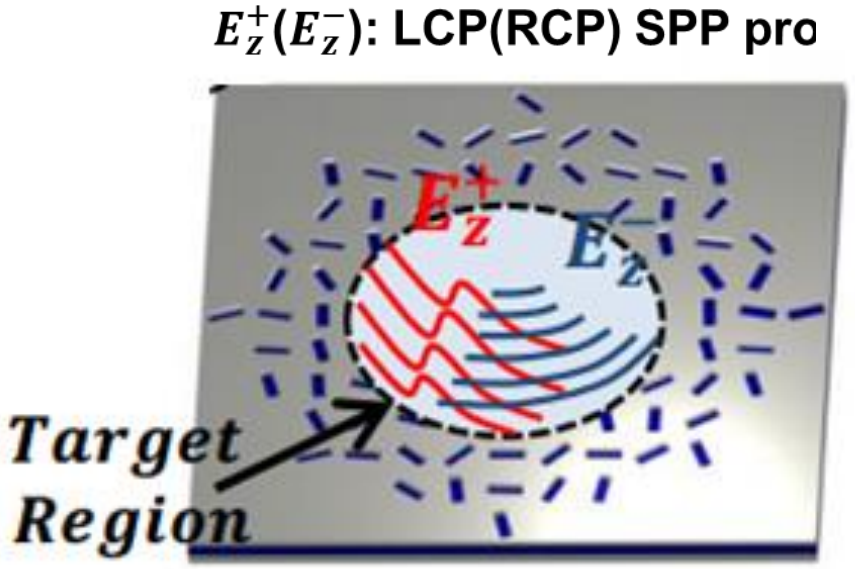
Orientation of slots = Holographic storage of SPP profiles



Experimental setup in generation SPP

Collaboration with Hui Liu's group

Our scheme: Holographic interface



Target profile
for a particular spin

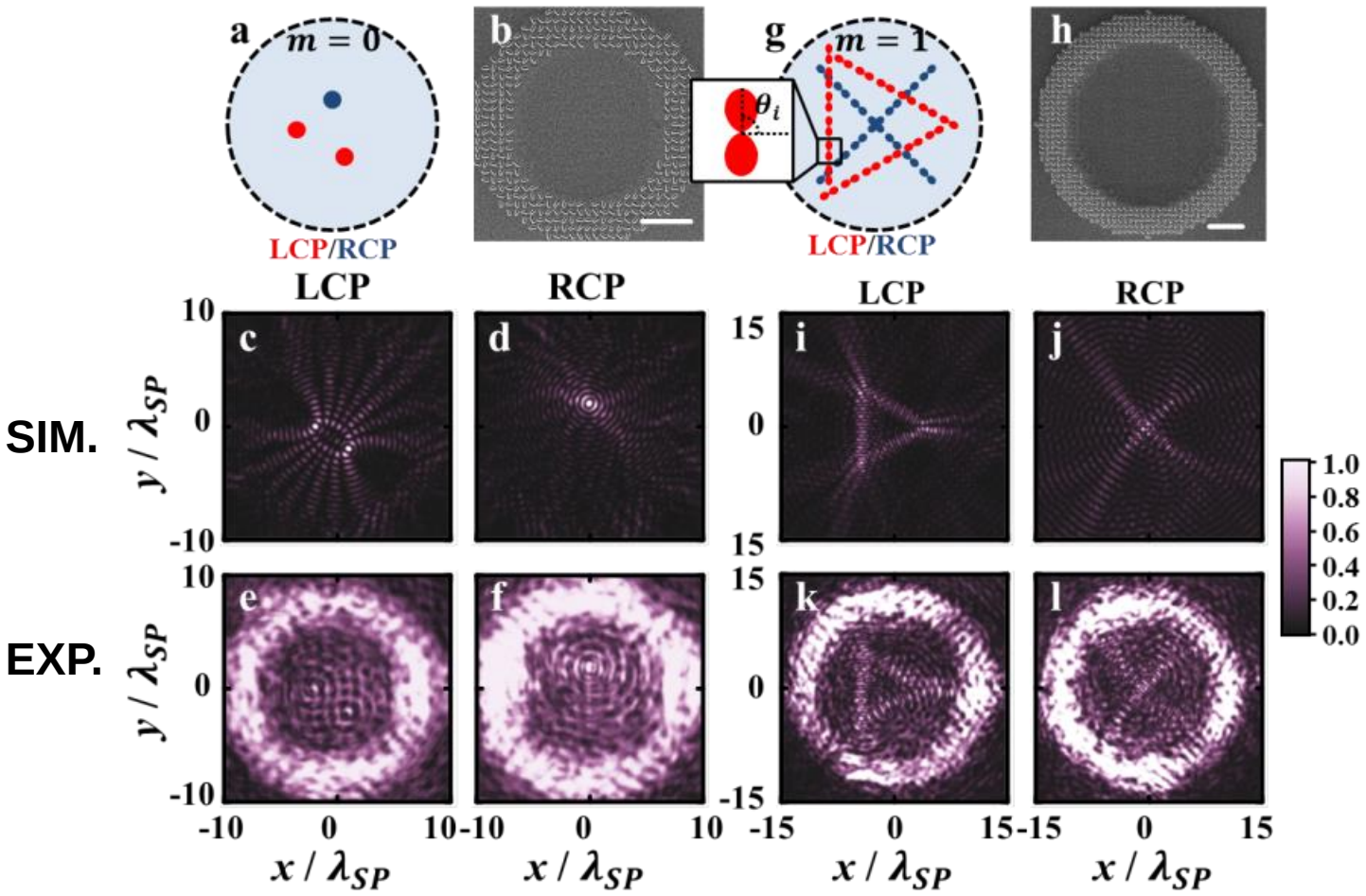
Geometric Matching Rule

$$f_{\pm}(E_z) = \pm\frac{1}{2}arg\left(\partial_x E_z \pm i\partial_y E_z\right) + \text{constant}$$

$$\alpha(x,y) = f_+\left(E_z^+ + (E_z^-)^*\right) = f_-\left(E_z^- + \left(E_z^+\right)^*\right)$$

- **Same set of atoms are responsible for both LCP and RCP profiles now**
- **Data capacity doubled**

Plasmonic Spin-Hall effect

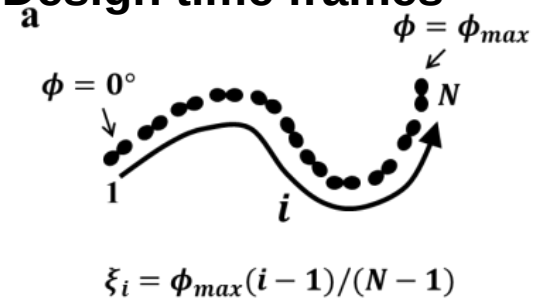


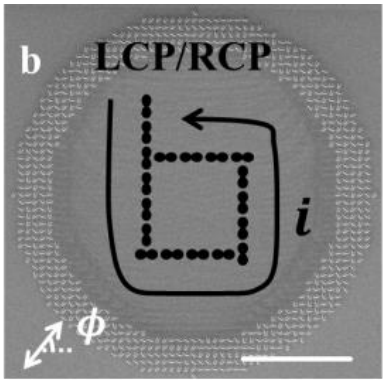
- **Spin-splitting of orbitals becomes arbitrary**
- **Opposite geometric phases of 2 spins from same set of atoms**

Nature Comm. 6 8360 (2015)

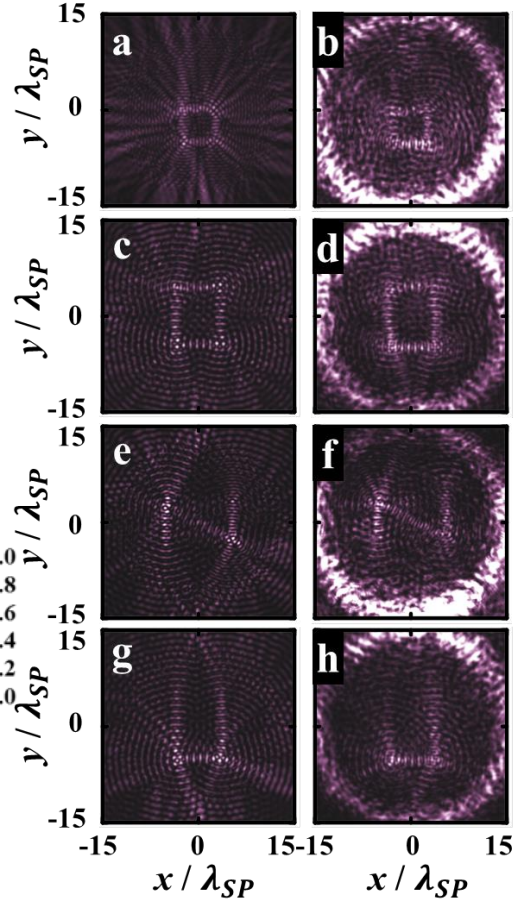
Coherent control for getting motion picture

Based on tunable pixels,
Design time frames



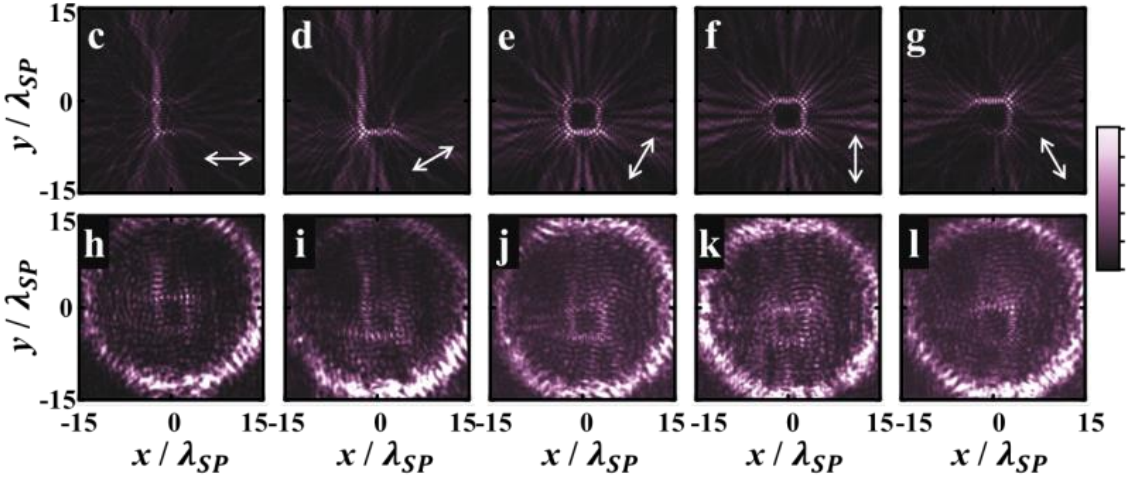


"Static" pictures



SIM.

EXP.



- Motion pictures played by polarization rotation angle
- The generated SPP profile becomes $cos(\phi) J_0(k_{SP}|r|)$

- **Duality can be restored by using metamaterials**
- **The geometric phase related can also be used to obtain plasmonic spin-Hall effect**

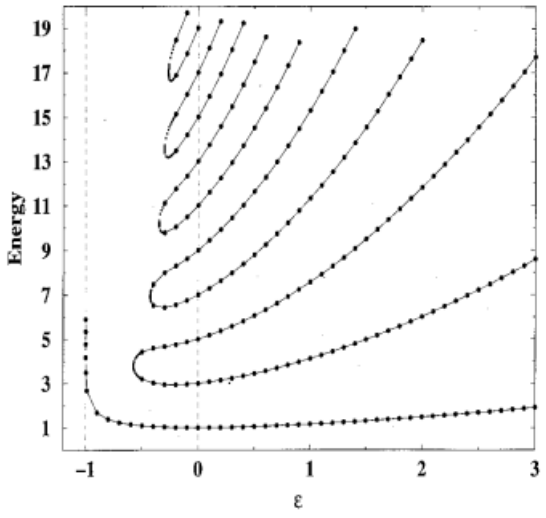
- **Next, to investigate the role of dual operation in constructing Parity-Time (PT) symmetric systems**
- **Physics enriched by controlling the symmetry breaking**

Parity-time symmetry in QM

- Quantum Mechanics
  - Observables are real numbers
  - General belief:
    - Real eigenvalues ⇔ Hermitian operators
- Parity-time (PT) symmetric Hamiltonian
  - A weaker condition can still give real eigenvalues
  - Phase transition (spontaneous symmetry breaking) between the real and complex phases using an external order parameter

$$H = p^2 + x^2 (ix)^{\varepsilon}$$

$$V(x) = V^*(-x)$$

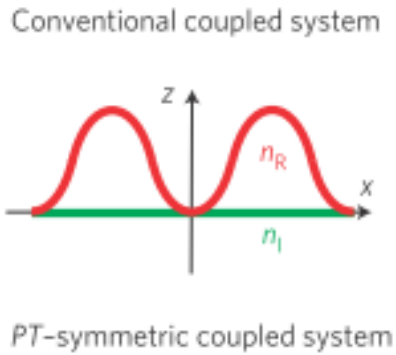


C. M. Bender @ Washington Univ., St. Louis

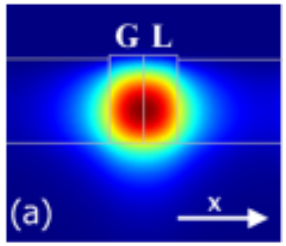
Parity-time symmetry in Optics

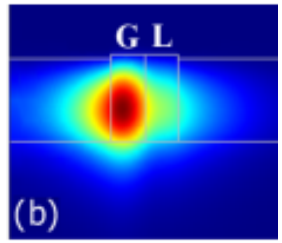
- PT-symmetric Hamiltonian can be established in optics
- Through a complex potential using ε(x)

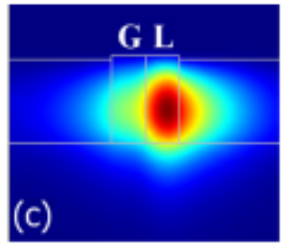
Pair of modes
with complex conjugated
Eigenvalues (propagation constants)

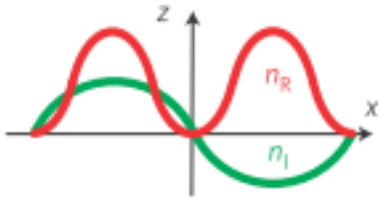


Real Eigenvalues





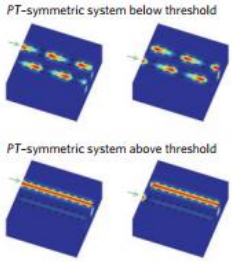




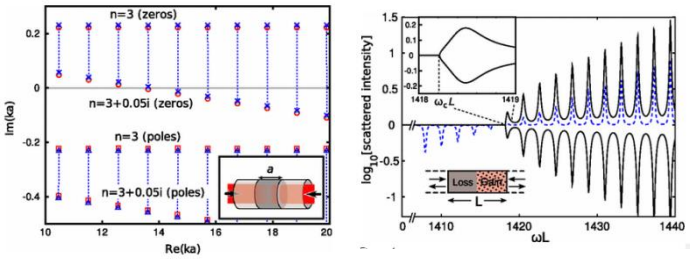
$$V(x) = V^*(-x)$$

A. Guo et.al., Phys. Rev. Lett. 103, 093902 (2009)
C. E. Rüter et.al., Nature Physics 6, 192 (2010)

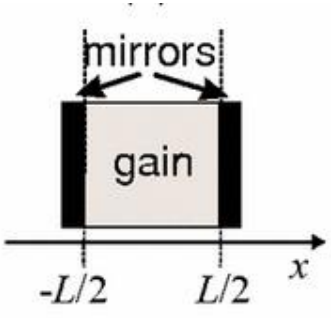
Parity–time symmetry in optics and metamaterials



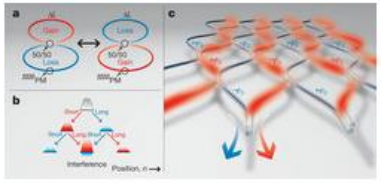
***PT* behaviour of a optical system**
C. E. Rüter *et. al.*
Nature Physics (2010)



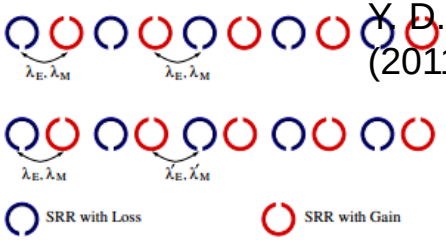
Coherent Perfect Absorbers
& PT phase transition
Y. D. Chong *et. al.* PRL 2010
Y. D. Chong *et. al.* PRL (2011)



PT-symmetric laser absorber
S. Longhi, PRA (2010)

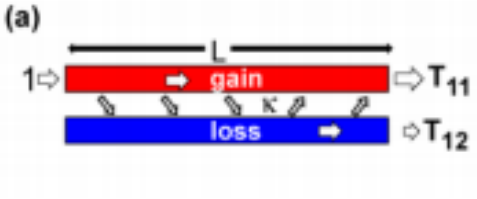


Parity–time synthetic photonic lattices
A. Regensburger *et. al.*
Nature (2012)

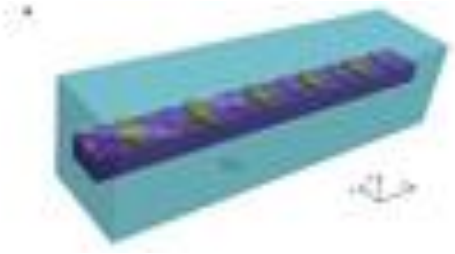


Gain-Driven Discrete Breathers

N. Lazarides and G. P. Tsironis
PRL (2013)



Switching in plasmonic system
A. Lupu et.al., OE (2013).



Experimental unidirectional reflectionless propagation
L. Feng et al. Nature Mater. (2013).

PT symmetry related phenomenon can be observed in optics and metamaterials

Establishing Hamiltonian with dual operation

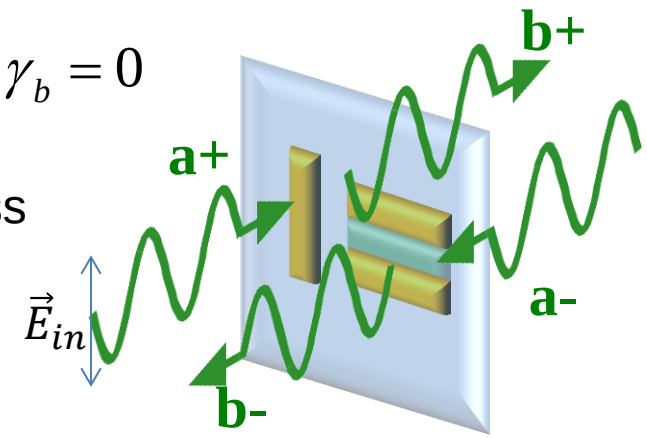
Absorption loss

➢1. Polarizing the atoms

$$\left(-\omega^2 - 2i\omega\left(\Gamma_a + \gamma_a\right) + \omega_0^2\right) p = 2 f_a (a_+ + a_- - i\omega g m),$$

$$\left(-\omega^2 - 2i\omega\Gamma_b + \omega_0^2\right) m = 2i\omega f_b g p, \quad \text{with } \gamma_a = f_a, \quad \gamma_b = 0$$

Scattering loss



(E-field amplitudes)

➢2. Scattered waves

$$a_+ - b_+ = a_- - b_- = -i\omega p$$

➢1. & 2. complete for us to get S-matrix

$$\begin{pmatrix} b_+ \\ b_- \end{pmatrix} = \mathbf{S} \begin{pmatrix} a_+ \\ a_- \end{pmatrix}$$

➢Linearization approx. $\omega_0^2 - \omega^2 \approx 2\omega(\omega_0 - \omega)$

➢Focus on phenomenon of coherent perfect approximation:

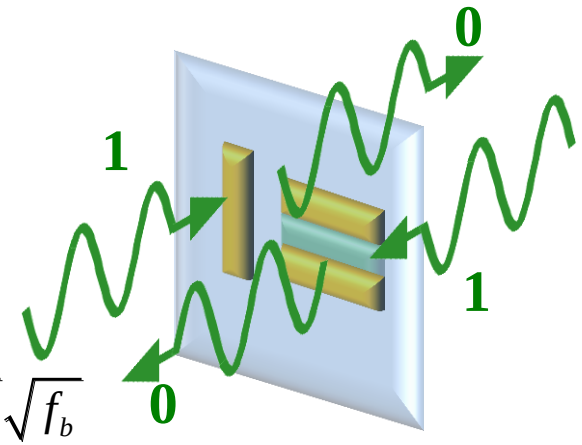
b+ = b- = 0

➢We get …

Establishing Hamiltonian with dual operation

- Coherent Perfect Absorption as an eigenvalue problem

$$\mathbf{S}\begin{pmatrix} a_+ \\ a_- \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow \mathbf{H}_2 \begin{pmatrix} \tilde{p} \\ \tilde{m} \end{pmatrix} = \omega \begin{pmatrix} \tilde{p} \\ \tilde{m} \end{pmatrix}$$



- where Hamiltonian is defined by

$$\mathbf{H}_2 = \begin{pmatrix} \omega_0 - i\left(\Gamma_a - \gamma_a\right) & i\kappa \\ -i\kappa & \omega_0 - i\Gamma_b \end{pmatrix}, \qquad \kappa = g\sqrt{f_a}\sqrt{f_b}$$

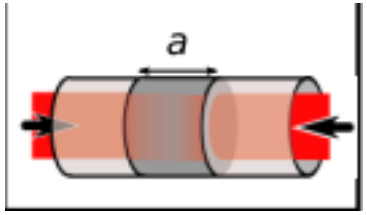
- Interpretation:
  - Renormalized (not $(\Gamma + \gamma)$) for normal mode consideration: incident waves as effective gain
  - Ideal PT-symmetry condition $\gamma_a - \Gamma_a = \Gamma_b$ (gain not needed)
  - Operators: $[PT, H_2] = 0$ with

$$P = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \text{ with } P^2 = I, \qquad T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} K$$

Phys. Rev. A 87, 053824 (2013).
CPA: Chong et.al., Phys. Rev. Lett. 105 (2010)

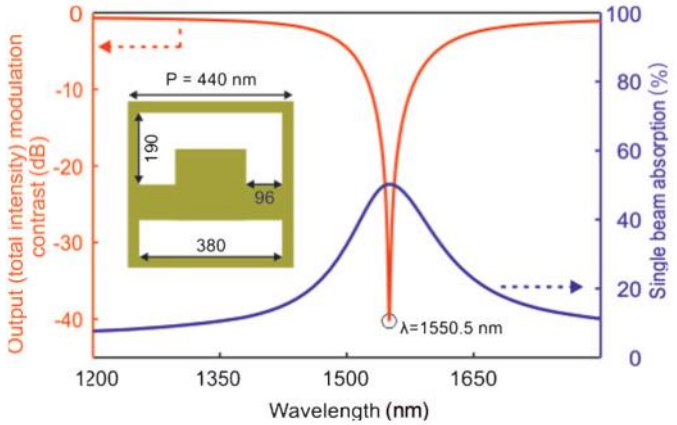
Coherent perfect absorption

- A way to modulate signal through interference of two beams of different directions

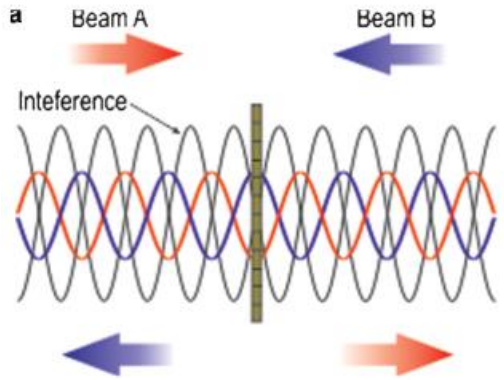


$$e^{inka} = \pm \frac{n - n_0}{n + n_0}.$$

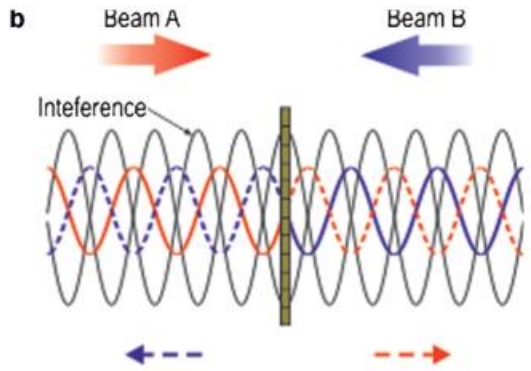
Coherent perfect absorber
Chong, Ge, Cao & Stone, PRL (2010)



Zhang, Macdonald & Zheludev, LSA (2012)



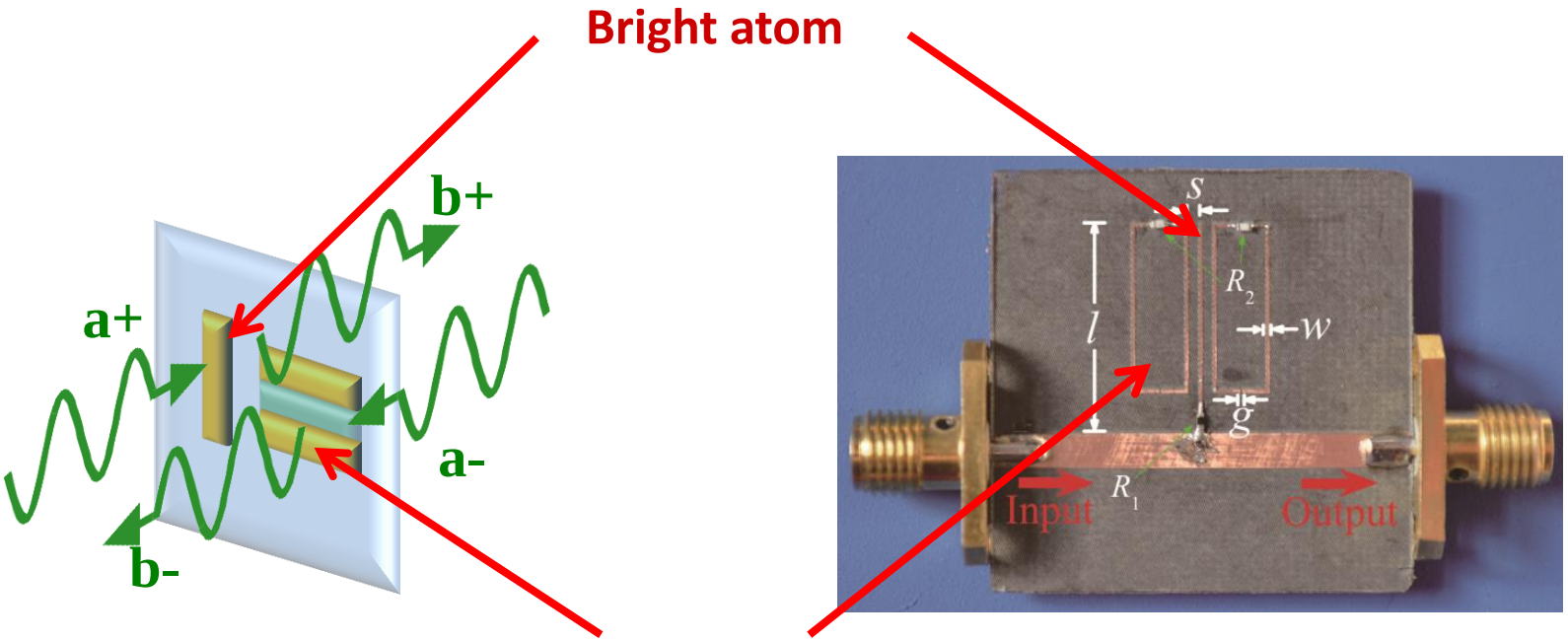
Out-of-phase: destructive inteference



in-phase: constructive inteference

Experimental demonstration

- Coupling between a bright and a dark atom
  - Probing PT-symmetry in conventional EIT config
  - But with intermediate coupling, comparable losses from 2 atoms



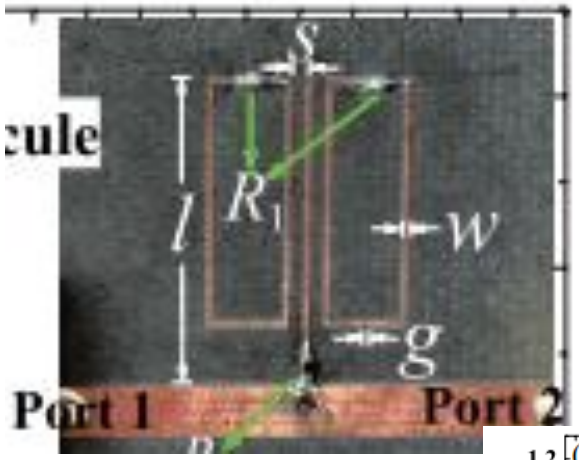
Metamaterial model configuration

Transmission-line analog

Experiment done by Hong Chen's group in Tongji University

Passive system realization

➢Extracting the dipolar model parameters from simulations



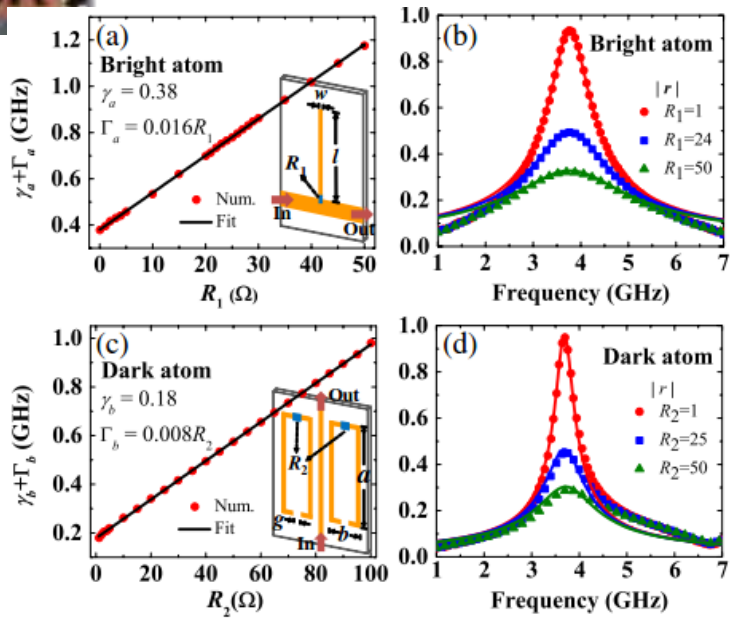
Fitting t/r spectrum

$$f_0 = 3.75GHz$$

$$\Gamma_a = 0.016R_1, \quad \gamma_a = 0.38GHz$$

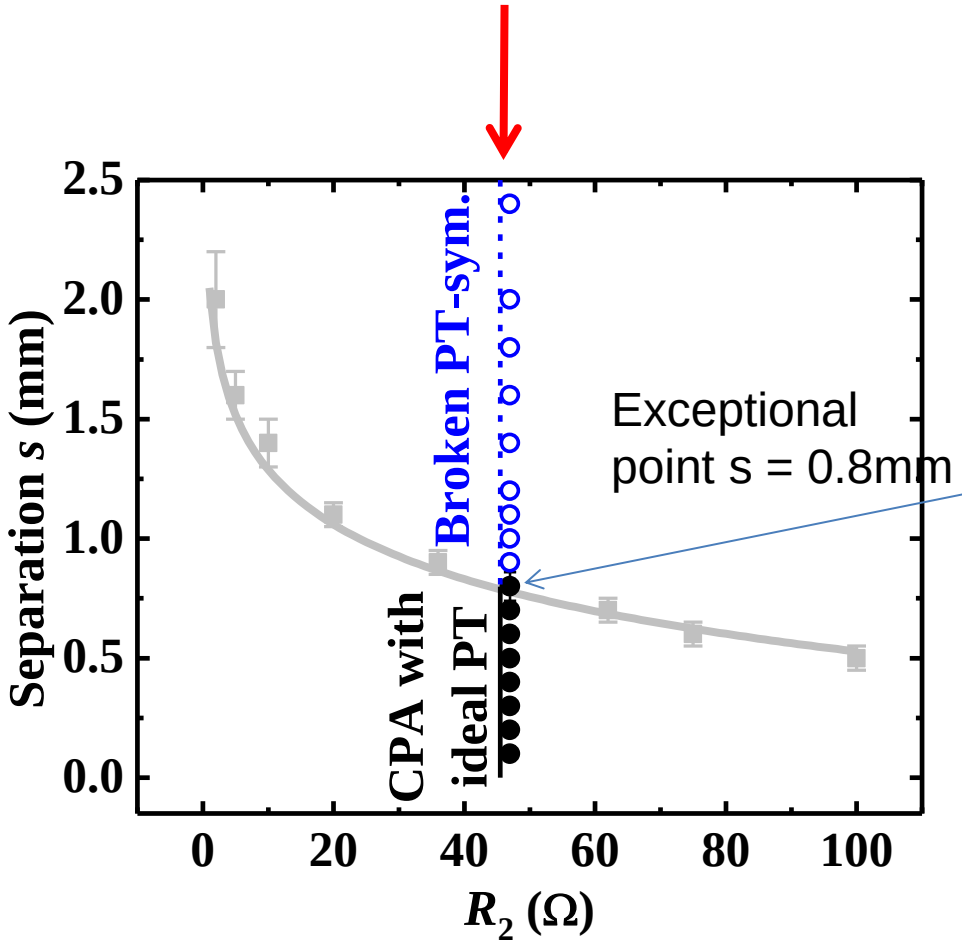
$$\Gamma_b = 0.008R_2, \quad \gamma_b = 0$$

$$\kappa = 1.2\exp\left(-s/0.66mm\right)$$

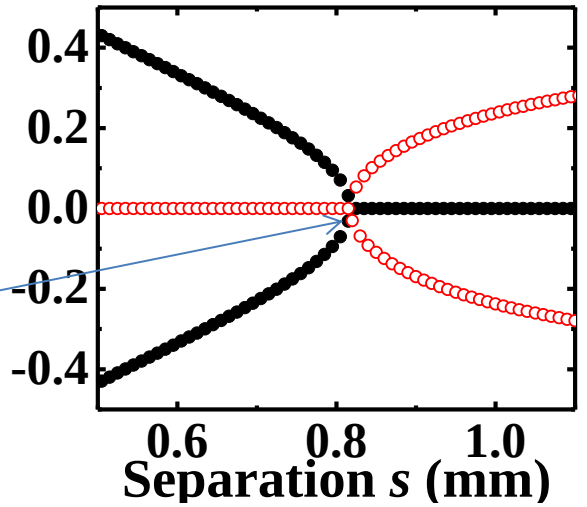


Spontaneous PT-symmetry breaking

- Ideal PT-symmetry: Black and blue curve
  - s (or equivalently κ) is varied
  - R1=1Ohm, R2=45.5 Ohm satisfying $\gamma_a - \Gamma_a = \Gamma_b$



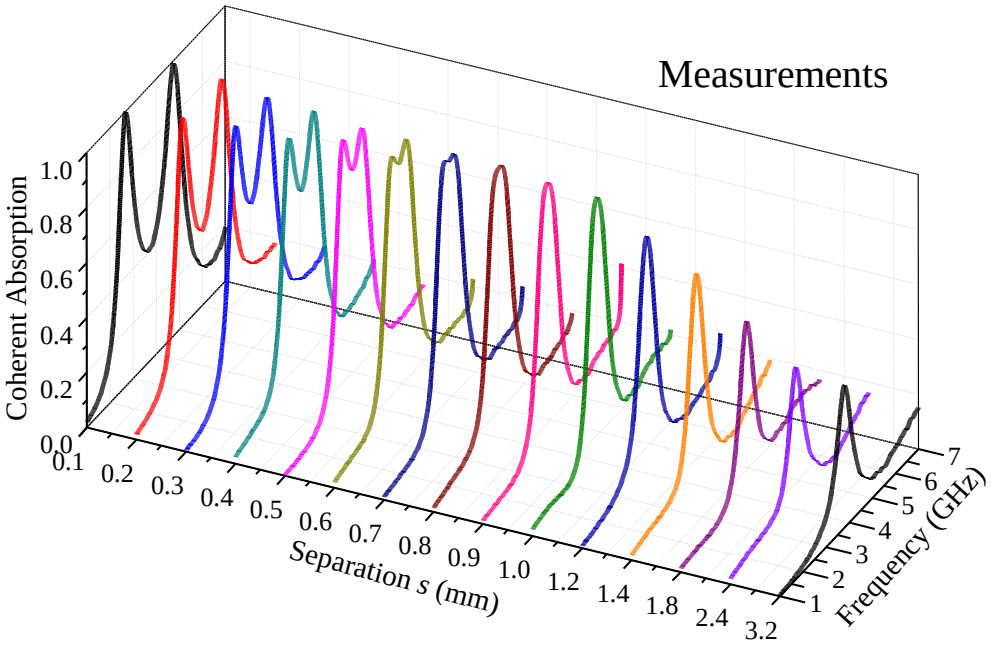
$$\mathbf{H}_2 = \begin{pmatrix} \omega_0 + i\Gamma_b & i\kappa \\ -i\kappa & \omega_0 - i\Gamma_b \end{pmatrix},$$



$$\omega = \omega_0 \pm \sqrt{\kappa^2 - \Gamma_b{}^2}$$

Spontaneous symmetry breaking

- PT-Phase transition between eiganvalues behavior
  - 2 absorption peaks (real eigenfrequencies) for s smaller than 0.8mm
  - Single absorption peak (complex conjugated eigenfrequencies) otherwise

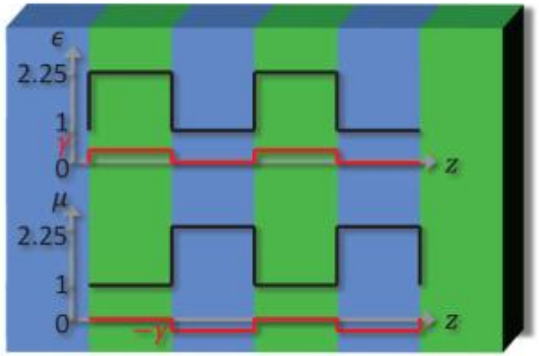


Phys. Rev. Lett. 112, 143903 (2014).

Establishing PT-symmetry for uni-directional reflectionless propagation

- Wave equation

$$\partial_z \begin{pmatrix} H_y \\ E_x \end{pmatrix} = ik_0 \begin{pmatrix} \epsilon(z) & 0 \\ 0 & \mu(z) \end{pmatrix} \begin{pmatrix} E_x \\ H_y \end{pmatrix}$$



- P-operator redefined

$$P \begin{pmatrix} \vec{E} \\ \vec{H} \end{pmatrix} = i\Pi M_z \begin{pmatrix} \vec{E} \\ \vec{H} \end{pmatrix}$$

$\Pi = \begin{pmatrix} 0 & \hat{z} \times I \\ -\hat{z} \times I & 0 \end{pmatrix}$ where $(\vec{E} \quad \vec{H})^* \Pi \begin{pmatrix} \vec{E} \\ \vec{H} \end{pmatrix}$ is the time-averaged power flow

  - Unitary, Hermitian, $P^2 = I$, symmetry operator of vacuum
  - Pairing mirror and dual operation

- T-operator

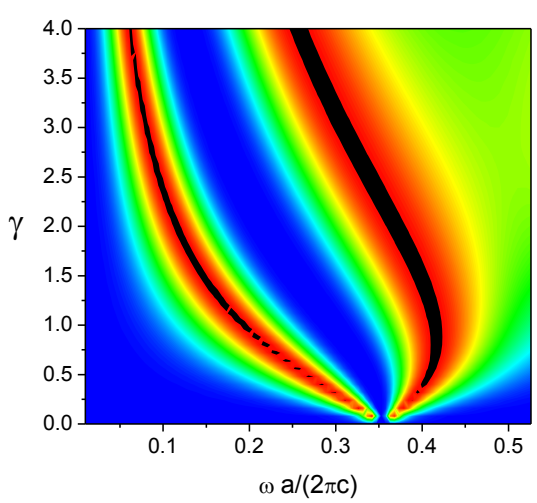
$$T \begin{pmatrix} \vec{E} \\ \vec{H} \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \begin{pmatrix} \vec{E}^* \\ \vec{H}^* \end{pmatrix}$$

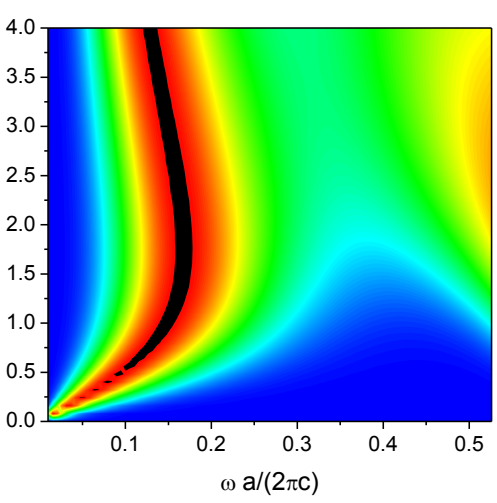
PT-symmetry condition

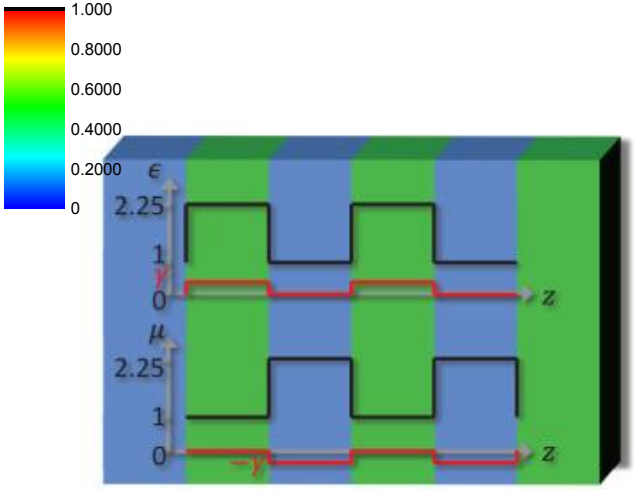
$$\epsilon(z) = \mu^*(-z)$$

Unidirectional reflection

- Merit = $\left(\left|\frac{R_f - R_b}{R_f + R_b}\right|\right)$
- Cross-potential matching: Unidirectional reflection at subwavelength thickness w/o requiring large gain/loss







Standard PT-matching

Cross-potential PT-matching

Phys. Rev. A 91, 033825 (2015)

Unidirectional reflection = Exceptional point of Hamiltonian

- Eigenvalue degeneracy of the S-matrix $S = \begin{pmatrix} t & r_b \\ r_f & t \end{pmatrix}$
- Equivalently the eigenvalue degeneracy of constitutive matrix
- Constitutive matrix interpreted as Hamiltonian (Hermitian for lossless) $B = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$$C^{-1} = \frac{2}{ik_0 a} B \frac{S - I}{S + I} B^{-1} = \begin{pmatrix} \epsilon_{\text{eff}} & i\kappa_{\text{eff}} \\ -i\kappa_{\text{eff}} & \mu_{\text{eff}} \end{pmatrix}$$

$$\begin{pmatrix} H_{y2} - H_{y1} \\ E_{x2} - E_{x1} \end{pmatrix} = \frac{ik_0 a}{2} C^{-1} \begin{pmatrix} E_{x2} + E_{x1} \\ H_{y2} + H_{y1} \end{pmatrix}$$



Inherited PT-symmetry condition

$\epsilon_{\text{eff}} = \mu_{\text{eff}}^*$ and $\kappa_{\text{eff}} = \kappa_{\text{eff}}^*$

- Advantage of cross-potential matching: gain and loss will not be averaged out to get back a Hermitian Hamiltonian

- **Using dual operation to establish PT-symmetric metamaterials**
- **A PT-phase transition of coherent perfect absorption can be observed using a totally passive system**
- **Unidirectional reflectionless propagation at subwavelength thickness can be observed**

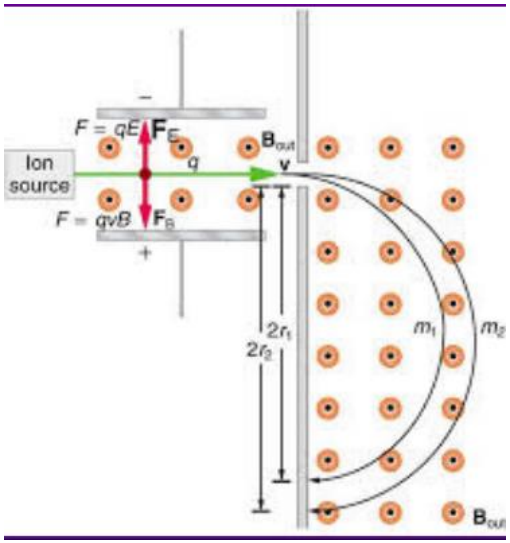
- **Next, we will consider a continuous vesion of dual operation to generate pseudo-magnetic field**

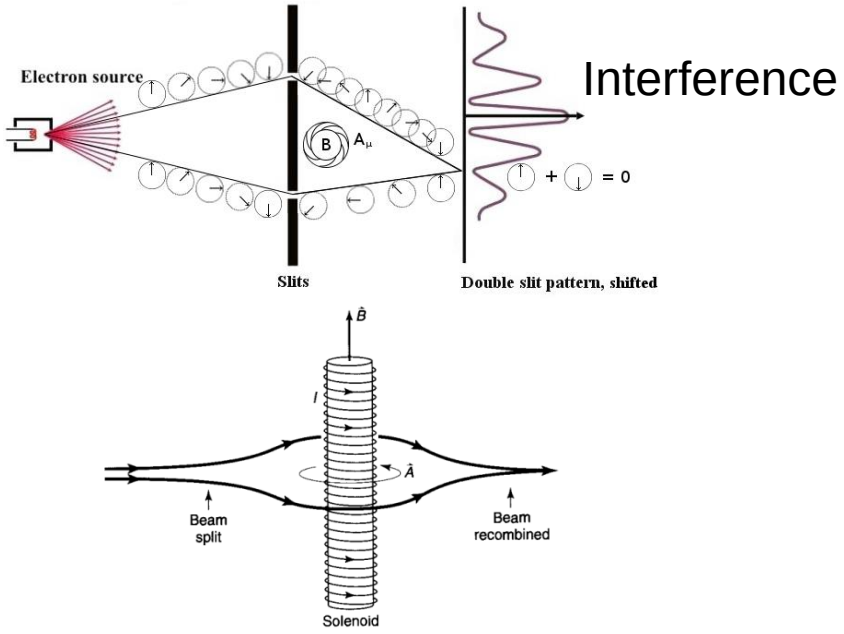
Magnetic field and gauge field for electron

$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ $\boldsymbol{B}$ unchanged under gauge transformation $\boldsymbol{A} \to \boldsymbol{A} + \boldsymbol{\nabla}\varphi$

- Magnetic field $\boldsymbol{B}$ bends charged particles (Hall effect)
- Lorentz force $q\boldsymbol{v} \times \boldsymbol{B}$

- Gauge field $\boldsymbol{A}$ modifies the phases (Aharonov-Bohm effect)
- Geometric phase $q/\hbar \int \boldsymbol{A} \cdot d\boldsymbol{r}$

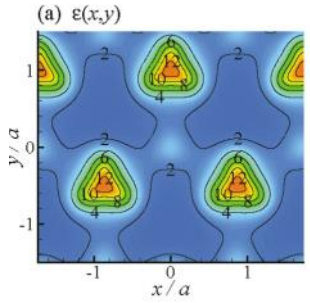


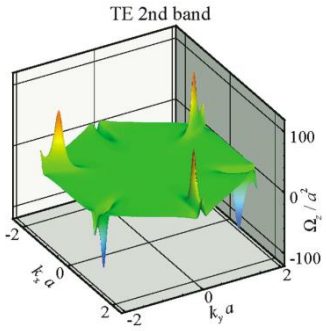


What about for photon?

Pseudo magnetic field for photon

- Pseudo magnetic field in ***momentum space***
  - anomalous velocity
  - giving Hall effect of light
  - beam displacement in the order of λ





Berry Curvature

$$\dot{\boldsymbol{r}}_c = \boldsymbol{v}(\boldsymbol{r}_c)\frac{\boldsymbol{k}_c}{k_c} + \dot{\boldsymbol{k}}_c \times (z_c|\boldsymbol{\Omega}_{\boldsymbol{k}_c}|z_c),$$

Berry Curvature

$$\dot{\boldsymbol{k}}_c = -[\boldsymbol{\nabla}\boldsymbol{v}(\boldsymbol{r}_c)]k_c,$$

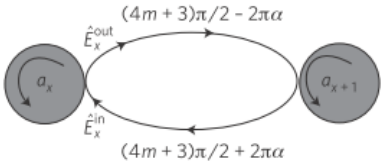
$$|\dot{z}_c) = -i\dot{\boldsymbol{k}}_c \cdot \boldsymbol{\Lambda}_{\boldsymbol{k}_c}|z_c),$$

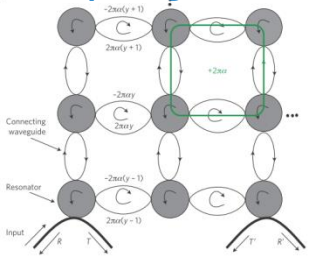
M. Onoda, S. Murakami, N. Nagaosa, *Phys. Rev. Lett*. **93**, 083901 (2004)
Bliokh et.al., Nature Photon. 2, 748 (2008)
Hosten et.al., Science 319, 787 (2008)

Pseudo magnetic field for photon

- Pseudo magnetic field in ***real space*** - lattice of resonators / waveguides in diffraction regime

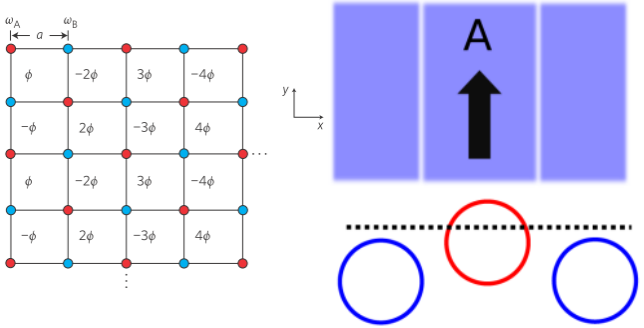
Direction-dependent coupling



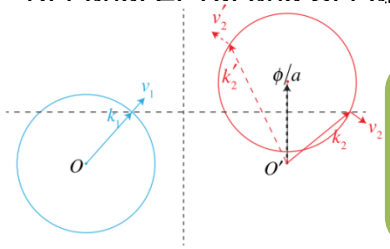


M. Hafezi et.al., *Nature Phys*. **7**, 907 (2011)

dynamic modulation (shifting of dispersion surface)

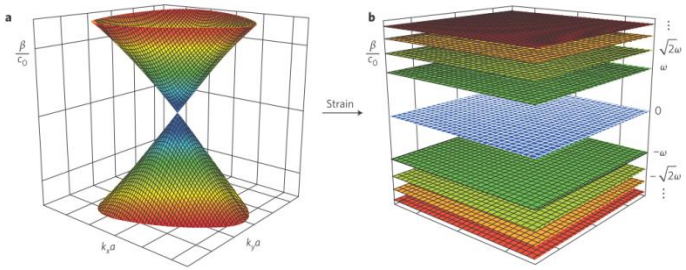


Q. Lin, and S. Fan, *Phys. Rev. X* **4**, 031031(2014)
K. Fang, and S. Fan, *Phys. Rev. Lett.* **111**, 203901(2013)
K. Fang, Z. Yu, and S. Fan, *Nat. Photonics* **6**, 782 (2012)



Additional way (to TO) to bend and guide light by magnetic Lorentz force

Strained photonic crystal



M. C. Rechtsman et.al., *Nature Photon*. **7**, 153 (2013)