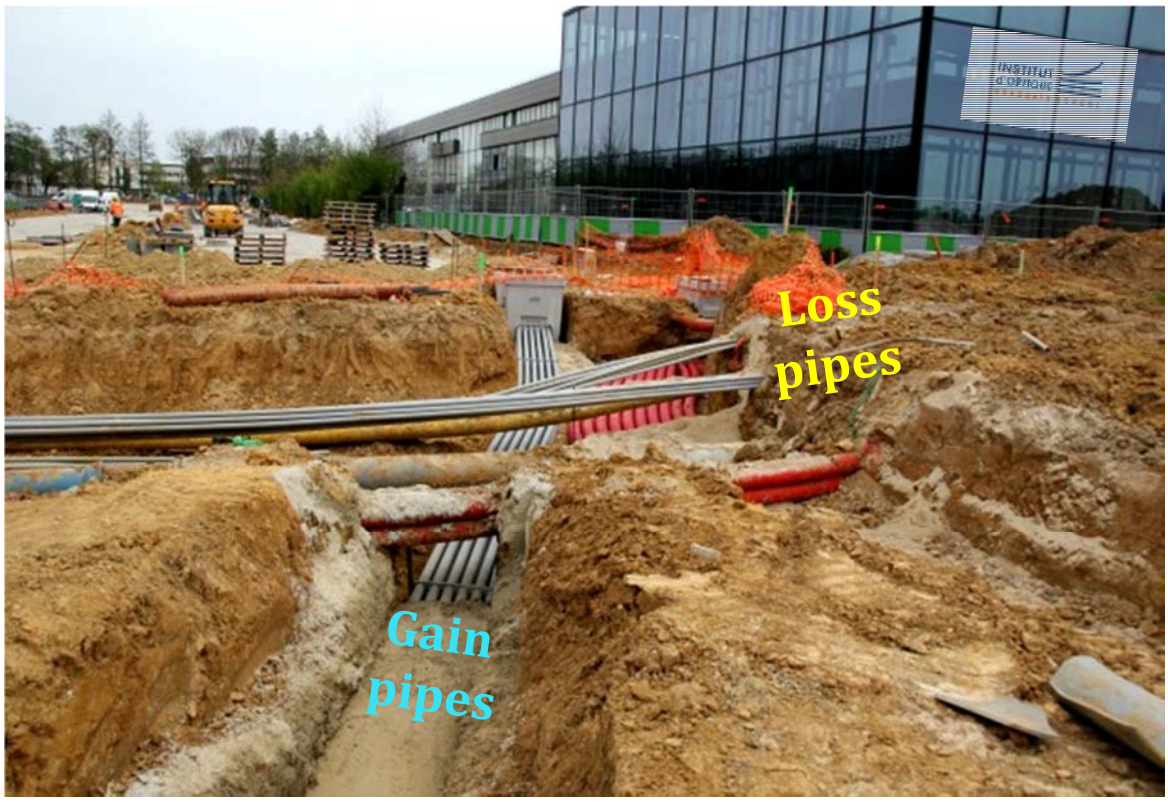
- Course 2 :
  - **PT –symmetry basics : the transverse case**
  - Imperfect PT–symmetry [→ plasmonics]
  - Application example :
    switching below the exceptional point

From here on :

Credits to : Anatole Lupu and Aloyse Degiron (**IEF**, Orsay, France)
Mondher Besbes and Jean-Paul Hugonin (my lab : **LCF**, IOGS, Palaiseau)

My inspiration ?



Quantum Mechanics usual postulates

- "The observables of a quantum system are defined to be the self-adjoint operators **A** on $\mathcal{H}$, a fixed complex Hilbert space."

$$< \mathbf{A}\Phi_j|\Phi_k > = < \Phi_j|\mathbf{A}\Phi_k >$$

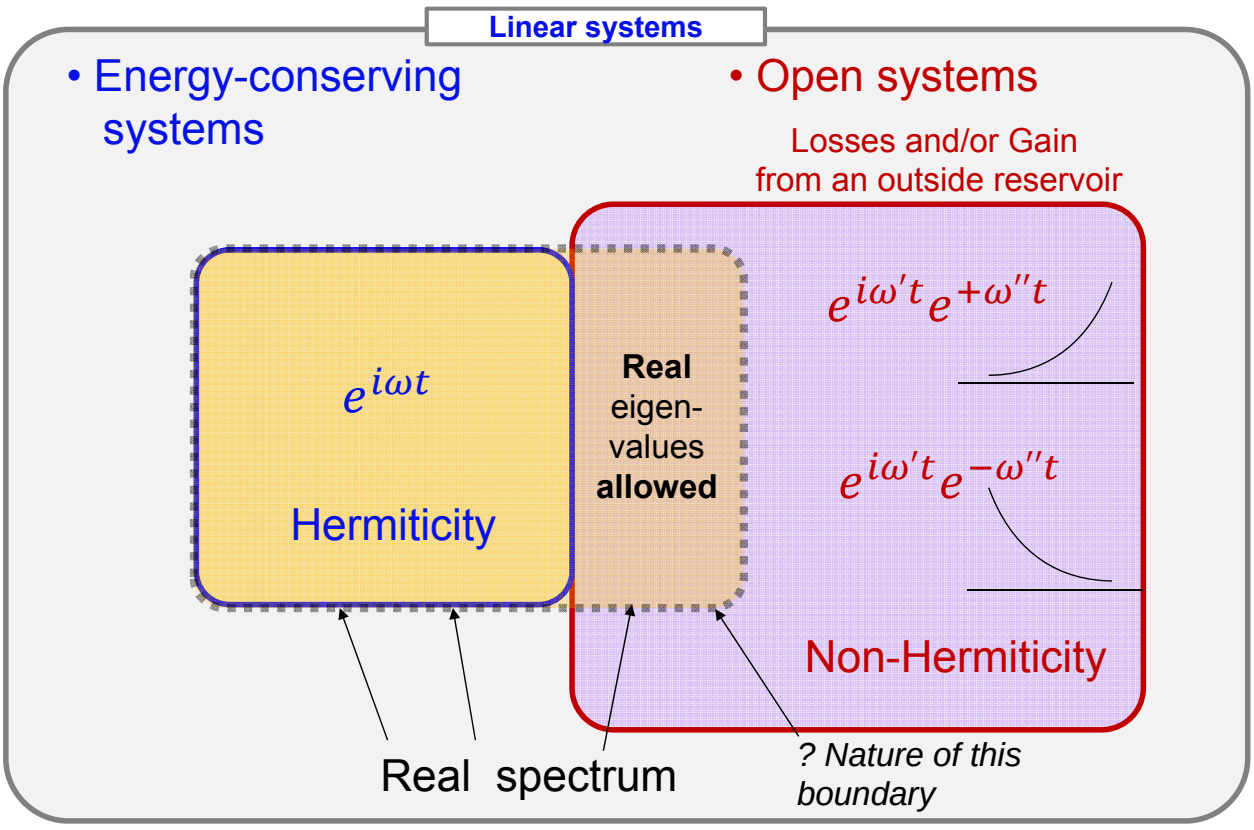
- "The Hamiltonian **H** is the operator associated to the total energy"

- In a finite-dimensional space $\{ |j> \equiv \Phi_j \}$,
  the Hamiltonian matrix
  $\mathbf{H} = \{H_{jk}\}$ is Hermitian : $H_{kj}{}^{*} = H_{jk}$

- This grants diagonal terms H_{jj} are real in any basis.
- Thus granting (eigen)energies $E_j = H_{jj}$ are real,
  using the basis of **H** eigenstates $\{ |j> \equiv \psi_j \}$, $\boldsymbol{H}\psi_j = E_j\psi_j$

" The spectrum is real"

The real spectrum



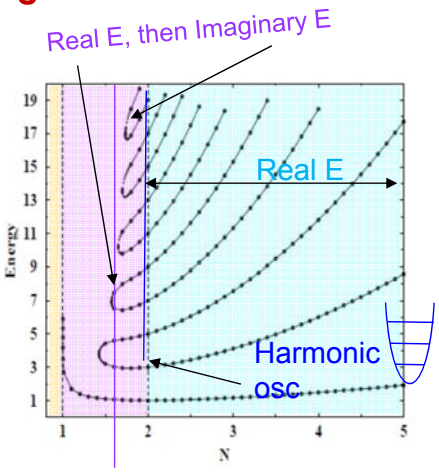
The real spectrum @ 1998

• Hermitian operator H ⇒ ♥ Real Eigenvalues

• Hermitian operator ⇏ ♥ Real Eigenvalues

C. M. Bender and S. Boettcher,
Phys. Rev. Lett. 80, 5243 (1998)

But Hamiltonians with 1D degree of freedom …
… are not that simple



Energy levels of the Hamiltonian $H = p^2 - (ix)^N$

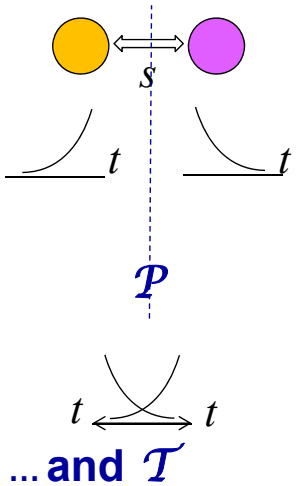
The simplest example "H"

$\mathcal{PT}$-Symmetric
Non-Hermitian 2x2 Hamiltonian

$$ih\frac{\partial}{\partial t}\psi = \begin{pmatrix} r'+ir'' & s \\ s & r'-ir'' \end{pmatrix}\psi$$

$\mathcal{P}$ $\mathcal{T}$

♥ Real Eigenvalues ... as long as $r'' < s$



... and $\mathcal{T}$

$$\det(H - E\mathbf{1}) = \begin{vmatrix} r'-E+ir'' & s \\ s & r'-E-ir'' \end{vmatrix}$$
$$= (r'-E)^2 + r''^2 - s^2$$

$\det(H - E\mathbf{1}) = 0 \rightarrow (r'-E)^2 = s^2 - r''^2$ whose sign is > 0 or < 0 !

$r'' < s$ or $r'' > s$

From QM to Optics (ca. 2007 for transv. geom.)

Analogy :
Time evolution in Quantum mechanics of Two-modes systems
⇕
Spatial evolution of two guided modes in z-propagation

$\mathcal{PT}$-Symmetric
Non-Hermitian Hamiltonian

$$ih\frac{\partial}{\partial t}\psi = \begin{pmatrix} r'+ir'' & s \\ s & r'-ir'' \end{pmatrix}\psi$$

$\mathcal{P}$ $\mathcal{T}$

♥ Real Eigenvalues ... as long as $r'' < s$

Coupled modes equations

$$i\frac{\partial}{\partial z}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \beta + ig & \kappa \\ \kappa & \beta - i\chi \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

gain losses

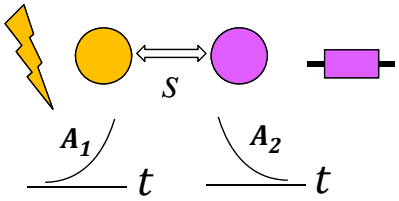
♥ Real Eigenvalues ... as long as $g < \kappa$

Gain → [$\mathcal{P}$ sym] → loss

$$\epsilon''(x) = -\epsilon''(-x)$$

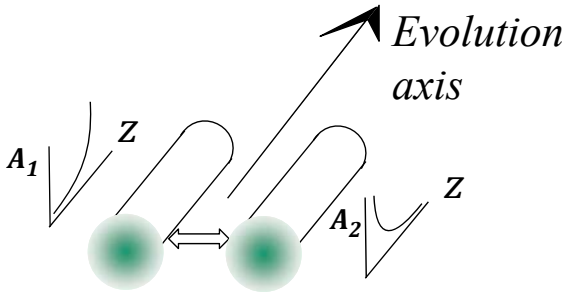
From QM to optics (visual)

Non conservative "atoms" (?!)(*)



(*) Gain → bosons

Non-conservative Guides



Evanescent coupling

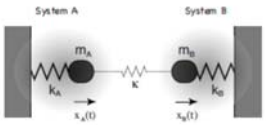
The topic of two coupled oscillators (classical)

Strong coupling, energy splitting, and level crossings: A classical perspective

Lukas Novotny[a)]
Institute of Optics, University of Rochester, Rochester, New York 14627

1199 Am. J. Phys. 78 (11), November 2010

$$\frac{\Gamma}{\gamma_A/m_A + \gamma_B/m_B} > 1.$$



PHYSICAL REVIEW A 77, 063833 (2008)

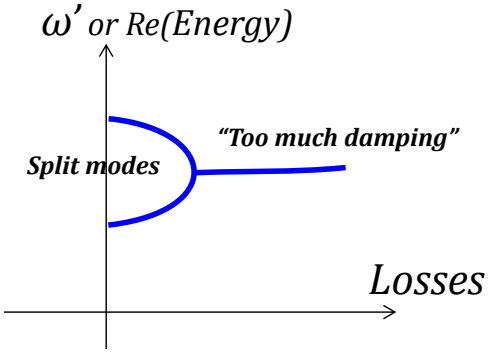
Spontaneous emission spectrum of a two-level atom in a very-high-Q cavity

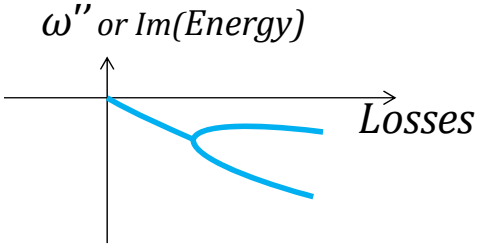
Alexia Auffèves,[1,*] Benjamin Besga,[1] Jean-Michel Gérard,[2] and Jean-Philippe Poizat[1]

Depending on the regime we consider, four experimental situations are possible as summarized in the following:

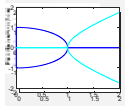
	Atomic-type spectroscopy	Cavity-type spectroscopy
Good emitter	*A*	*B*
Good cavity	*C*	*D*

(14)

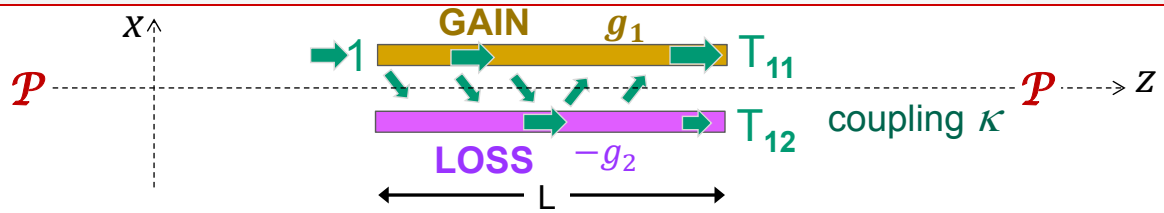




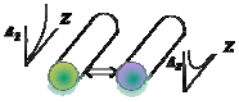
Change of behaviour in eigenvalues ...
... but in high-loss regime

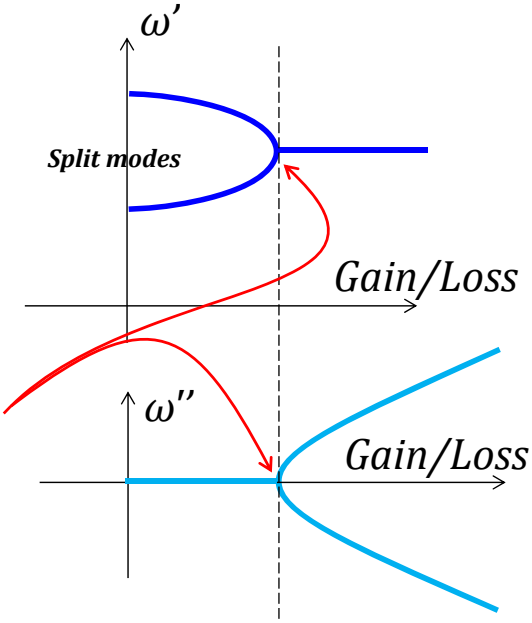


The case of balanced gain/loss



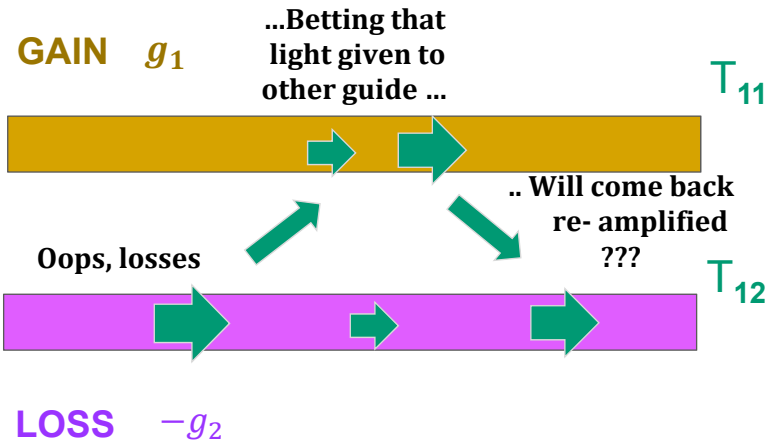
$$\epsilon'(x) = +\epsilon'(-x)$$
$$\epsilon''(x) = -\epsilon''(-x)$$





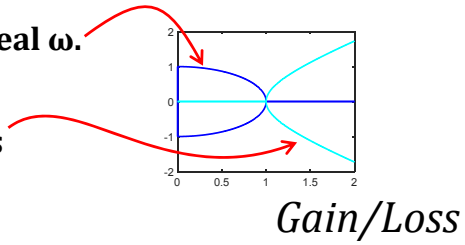
Exceptional point (EP) : singularity of ∂(eigenvalues)/∂(gain)

The light's bet



Bet works ? → Below EP, oscillation, real ω.

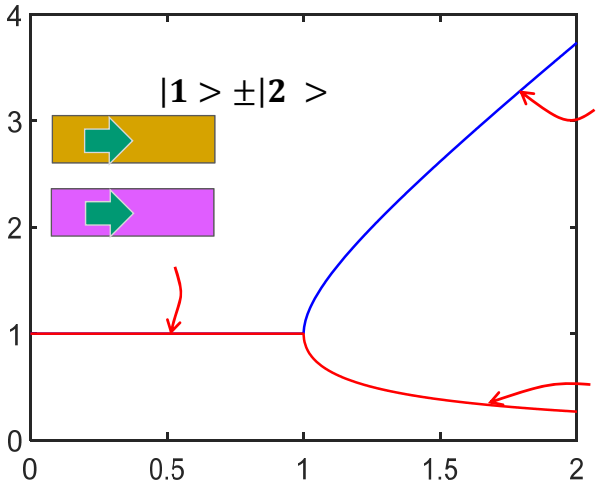
Bet lost ? → overall gain or loss ?
What are eigenstates



Gain/Loss

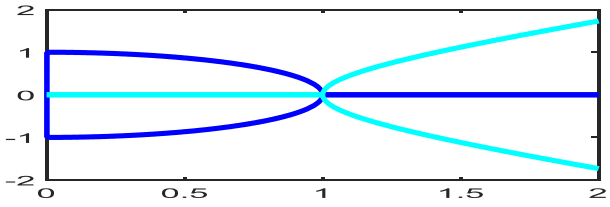
Eigenstates below/above EP

$|c_1/c_2|$



$|1> -c|2>$

$c|1> +|2>$

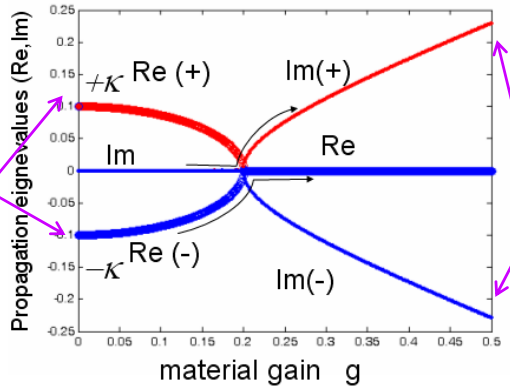


Gain/Loss

eigenstate behaviour vs. « gain-loss »

Symmetry breaking, pictorially

$$\frac{1}{\sqrt{2}}\begin{pmatrix} A \\ \pm A \end{pmatrix}$$

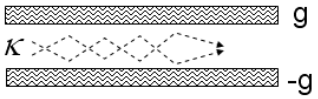


$$\begin{pmatrix} A \\ B \end{pmatrix}$$

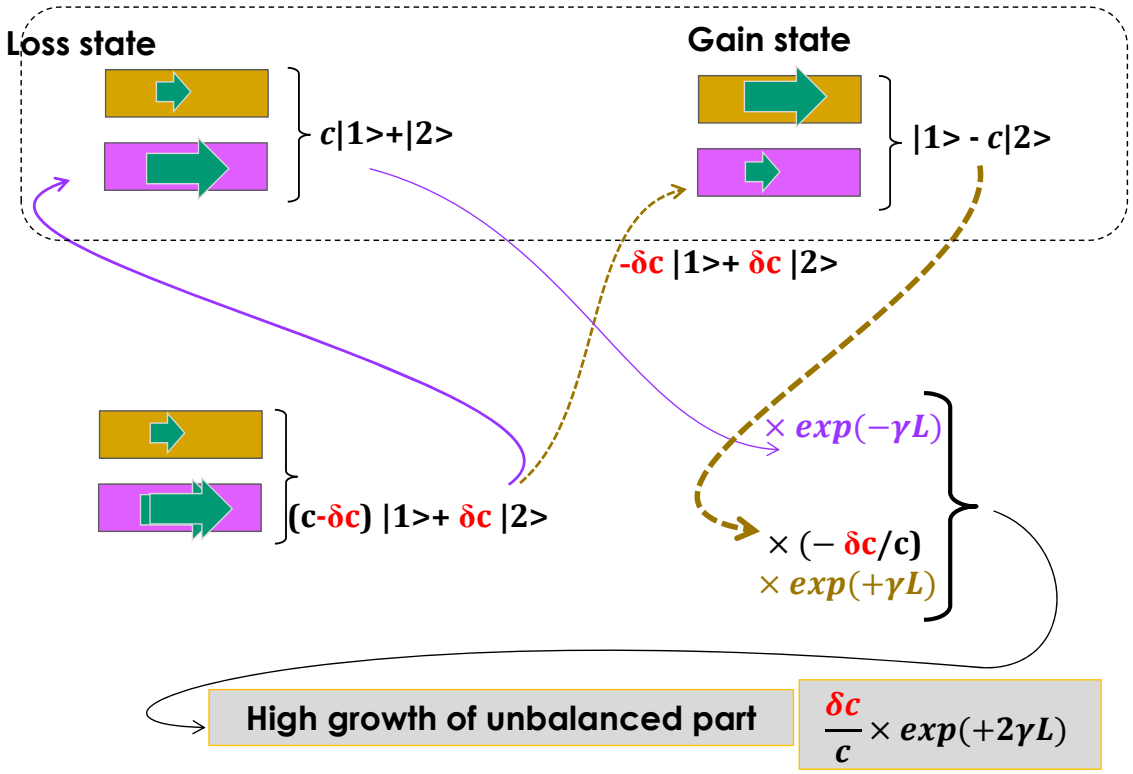
- Symmetry-breaking of eigenstates

$$\begin{pmatrix} A \\ B \end{pmatrix}$$

("winner-takes-all")

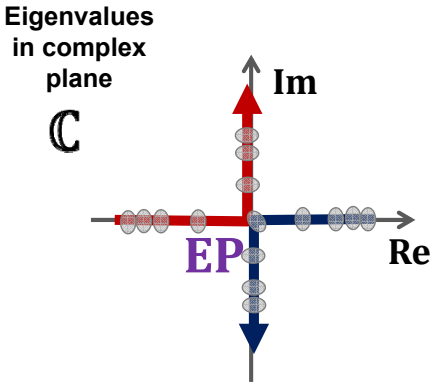


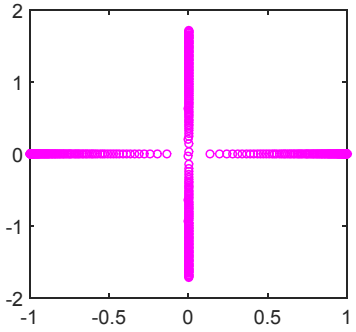
Danger of unbalanced eigenstates with gain/loss

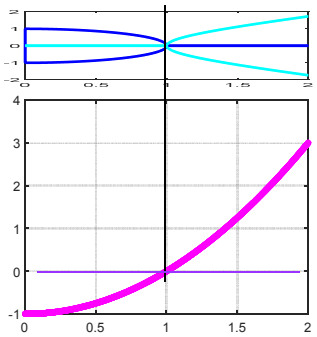


Can be misleading (Reciprocity based on exact eigenmodes)

Trajectory of eigenvalues in complex plane







Note (tracking tool for real wg, hint from *J-P Hugonin*):

Product of eigenvalues $\Pi=(\omega_1\,\omega_2)$ is continuous at EP!
Sum $\Sigma=(\omega_1+\omega_2)$ continuous as well

$\mathcal{PT}$ symmetry in Optics

« unnamed » *(& very strong κ)*

• Ctyroky & Nolting 1996

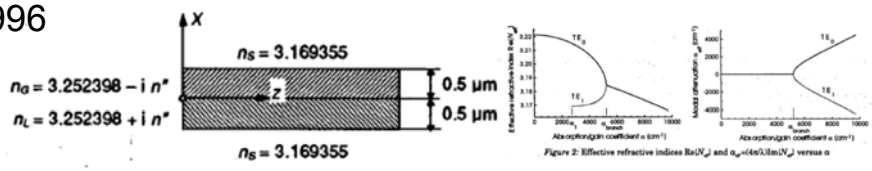


Figure 1: Two-layer waveguide with gain and loss

• 2004-2005 : Kulishov/Greenberg/Poladian/Agarwal: ***(tomorrow)***

Gratings with $\Delta n = \Delta n_r \cos(Kz) + i\Delta n_i \cos(Kz+\varphi)$, *««««nonreciprocity»»»»*

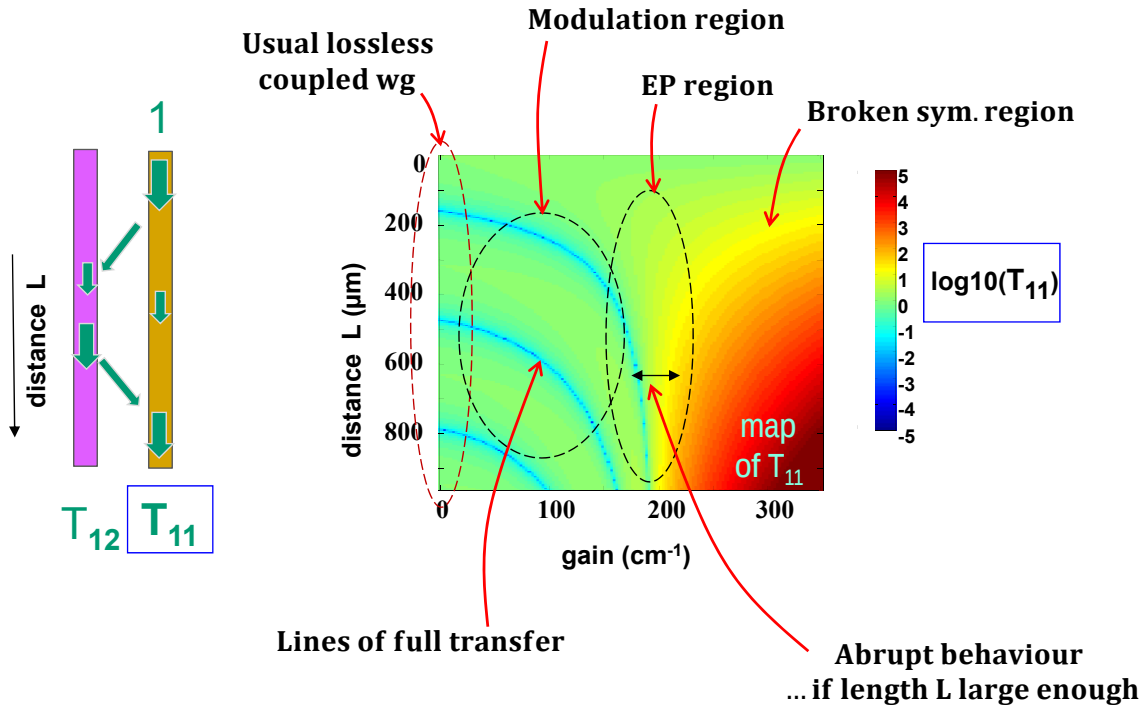
"named"

• El Ganainy et al. (CREOL),
« **Theory of coupled optical *PT*-symmetric structures** », Opt. Lett. **32**, 2632 (2007)
• Klaiman et al. PRL 2008; Guo et al. PRL 2009; … (topic starts to blow up)

Observed with parametric gain/loss

• Rüter et al. (Clausthal u. CREOL, Technion)
« **Observation of parity–time symmetry in optics** », Nat. Phys. **6**, 192 (2010)

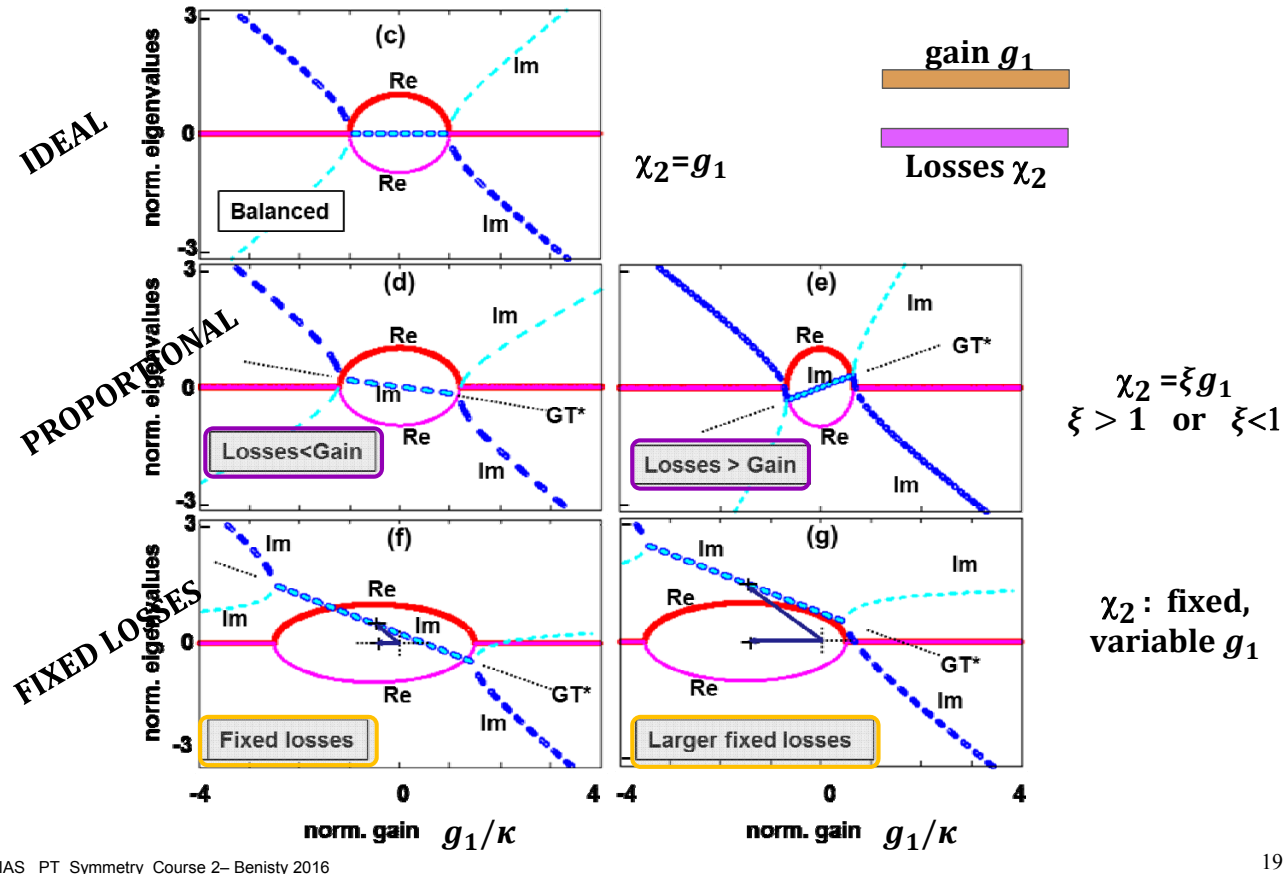
The overall behaviour (Transmission matrix)



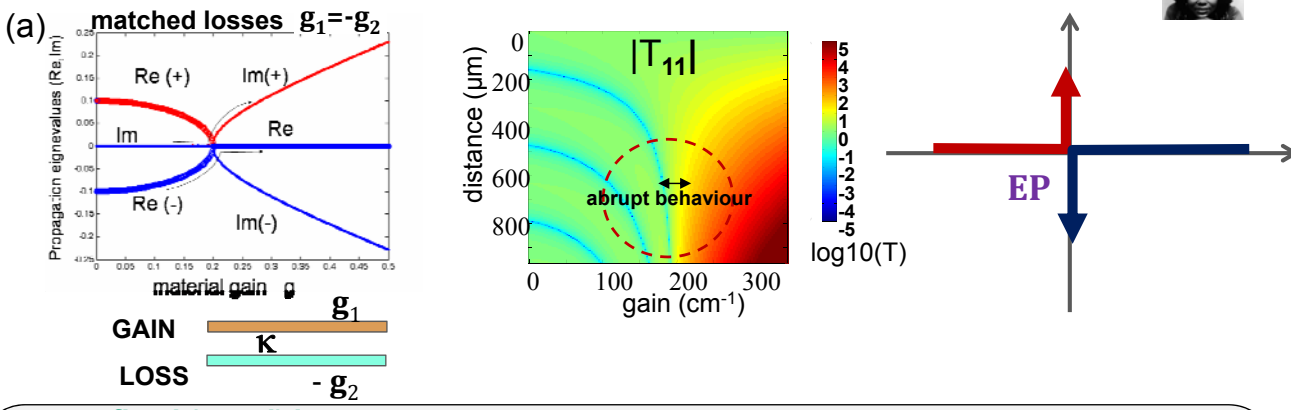
PT-symmetry and Waveguides / (2) The transverse case

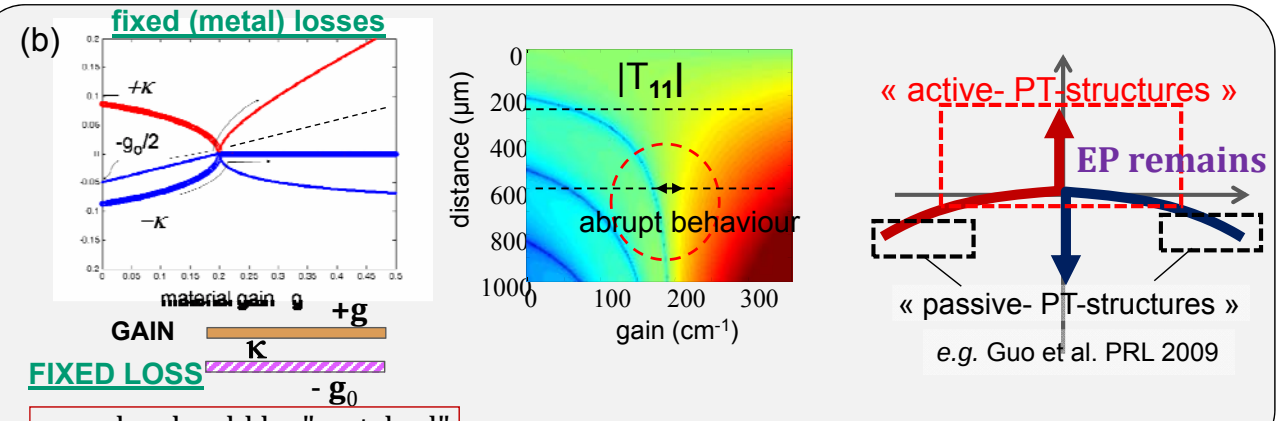
- Course 2 :
  - PT –symmetry basics : the transverse case
  - **Imperfect PT–symmetry [→ plasmonics]**
  - Application example :
    switching below the exceptional point

3 FLAVOURS OF IMPERFECT $\mathcal{PT}$-SYMMETRY



Fixed losses : Survival of abrupt transition

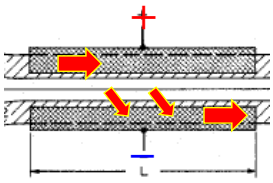


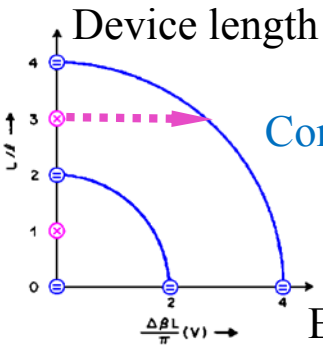


H. Benisty et al. *Opt. Express*, **19**, *18004, 2011*

$\mathcal{PT}$–symmetry & plasmonics

Kogelnik 1970's





PT-sym, 2010's

Gain / no gain

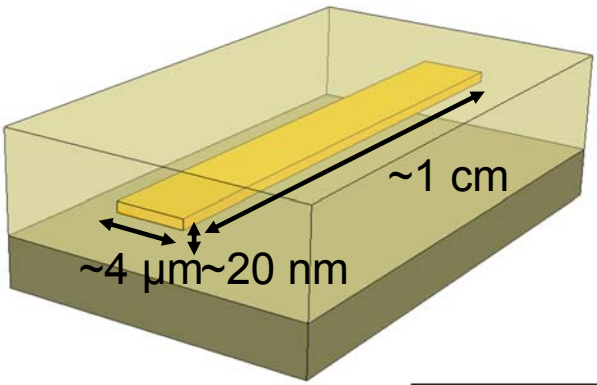
Fixed losses (Au)

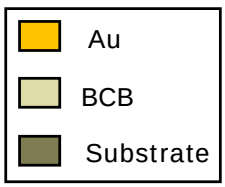
Very few tunable ΔRe(***n***) proposals in plasmonics

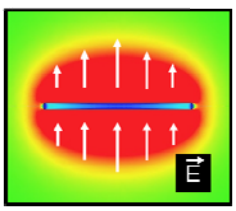
→ **We'd better tune Δ*Im*(*n*) !**

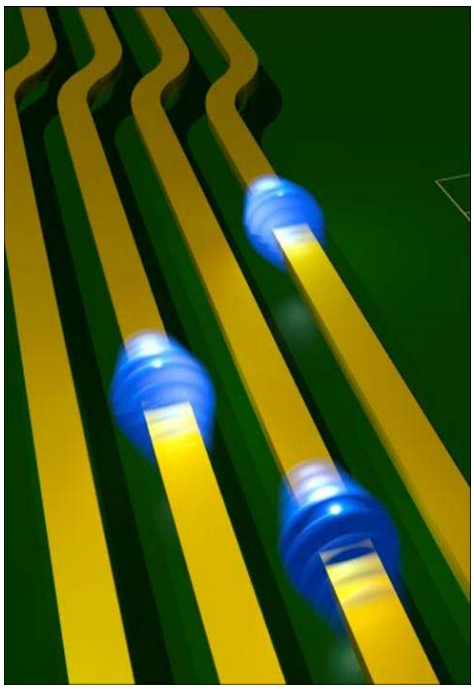
?? **What are the new diagrams ?**

Long – Range Surface Plasmon (LRSPP)





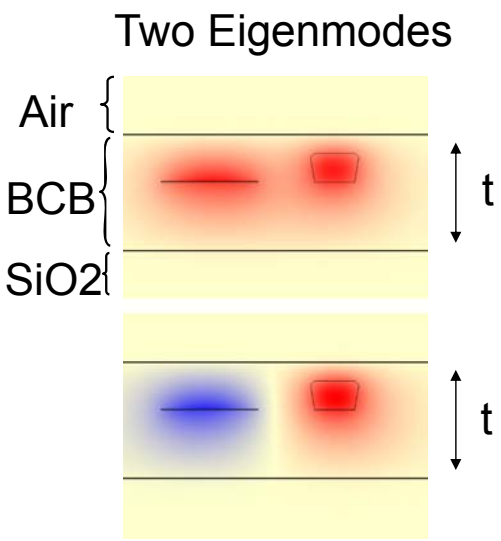




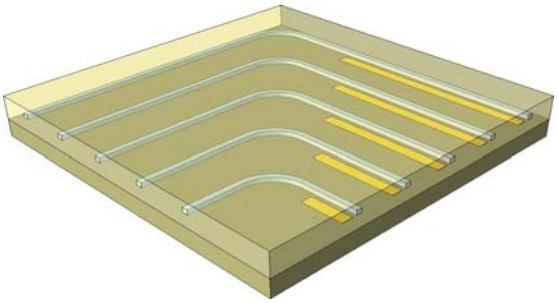
Wideband mode (VIS-IR range)

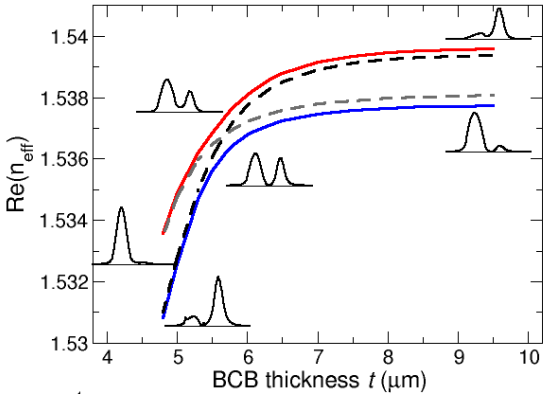
Courtesy of A. Degiron

Co-directional coupling between plasmonic guide and dielectric guide



Degiron et al, New J. Phys. 2009 (Duke U)



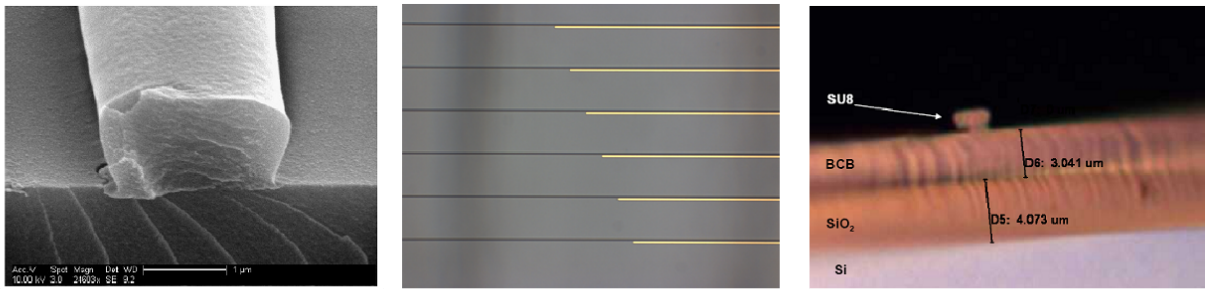


Detuning control by BCB thickness

Experimental waveguides

A combination of negative and positive lithography steps are used to fabricate plasmonic stripes coupled to SU8 waveguides, embedded in BCB polymer.

(a)



(b)

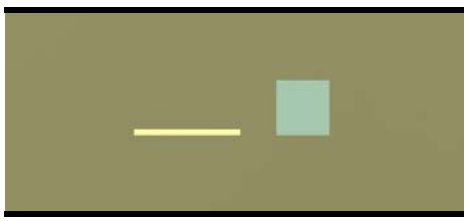


CL = coupling length

Measurements vs. coupling length

Arbitrary BCB thickness

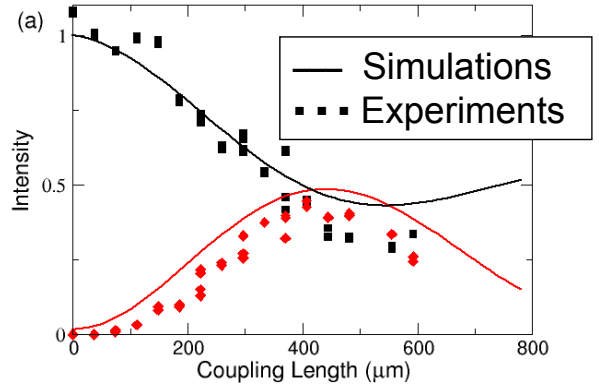
BCB
t=6.6 μm

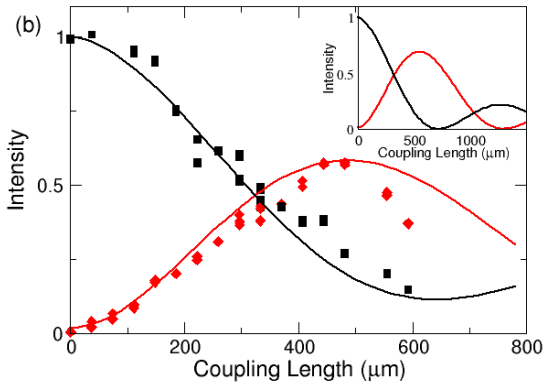


Optimized BCB thickness

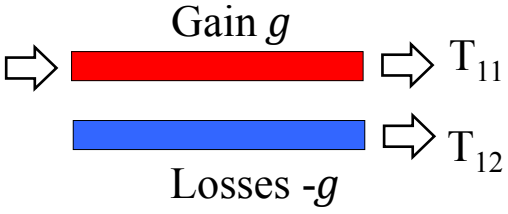
BCB
t=5.4 μm







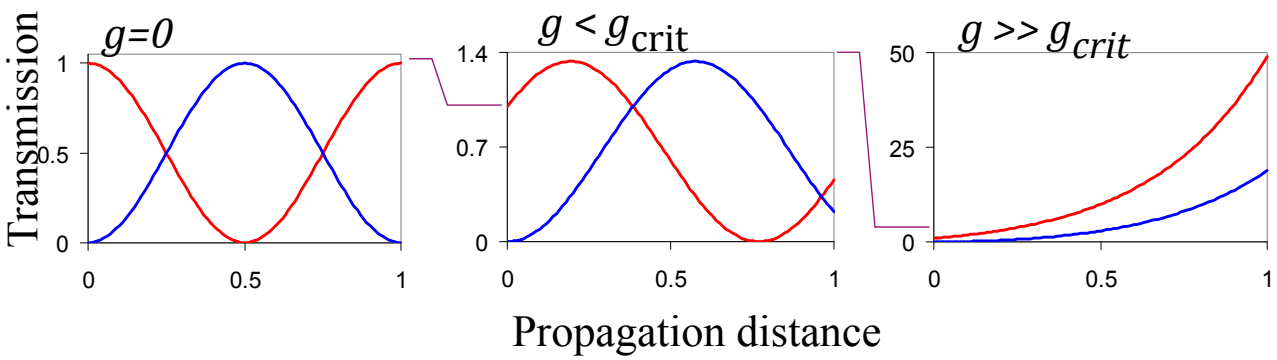
Dephasing of beating is a first sign



Coupled Mode Theory

$$i\frac{d}{dz}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=M\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$M=\begin{bmatrix}\beta+ig/2 & \kappa\\ \kappa & \beta-ig/2\end{bmatrix}$$



- Course 2 :
  - PT –symmetry basics : the transverse case
  - Imperfect PT–symmetry [→ plasmonics]
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    switching below the exceptional point

The exact transfer matrix of the arbitrarily detuned case

$$\frac{\partial}{\partial z}\begin{pmatrix}A_1\\A_2\end{pmatrix} = i\begin{pmatrix}\beta_1 + ig_1 & \kappa\\ \kappa & \beta_2 - i\chi_2\end{pmatrix}\begin{pmatrix}A_1\\A_2\end{pmatrix}$$

$\Delta_{\mathrm{im}} = \dfrac{g_1 + \chi_2}{2}$ Im detuning

$\delta = \dfrac{1}{2}[\beta_1 + ig_1 - (\beta_2 - i\chi_2)] = \dfrac{1}{2}[\beta_1 - \beta_2] + i\Delta_{\mathrm{im}}$ complex detuning

$\Omega = \sqrt{\delta^2 + \kappa^2}$ detuning & coupling

$\begin{pmatrix}A_1(z)\\A_2(z)\end{pmatrix} = M(z)\begin{pmatrix}A_1(0)\\A_2(0)\end{pmatrix}$ Transfer matrix

Solution

$$M(z) = \begin{pmatrix}\cos(\Omega z) - \dfrac{i\delta}{\Omega}\sin(\Omega z) & \dfrac{i\kappa}{\Omega}\sin(\Omega z)\\ \dfrac{i\kappa}{\Omega}\sin(\Omega z) & \cos(\Omega z) + \dfrac{i\delta}{\Omega}\sin(\Omega z)\end{pmatrix}\exp\left((g_1 - \chi_2)\frac{z}{2}\right)$$

(Kogelnik 1976)

The exact transfer matrix of Im-only detuning case

$$\frac{\partial}{\partial z}\begin{pmatrix}A_1\\A_2\end{pmatrix}=i\begin{pmatrix}\beta_0+ig_1 & \kappa\\ \kappa & \beta_0-i\chi_2\end{pmatrix}\begin{pmatrix}A_1\\A_2\end{pmatrix}$$

$\Delta_{\mathrm{im}}=\dfrac{g_1+\chi_2}{2}$ Im detuning

$\delta=+i\Delta_{\mathrm{im}}$ complex detuning

$\Omega=\sqrt{-\Delta_{\mathrm{im}}{}^2+\kappa^2}$ detuning & coupling

$\begin{pmatrix}A_1(z)\\A_2(z)\end{pmatrix}=M(z)\begin{pmatrix}A_1(0)\\A_2(0)\end{pmatrix}$ Transfer matrix

Simple solution

$$M(z)=\begin{pmatrix}\cos(\Omega z)+\dfrac{\Delta_{\mathrm{im}}}{\Omega}\sin(\Omega z) & \dfrac{i\kappa}{\Omega}\sin(\Omega z)\\ \dfrac{i\kappa}{\Omega}\sin(\Omega z) & \cos(\Omega z)-\dfrac{\Delta_{\mathrm{im}}}{\Omega}\sin(\Omega z)\end{pmatrix}\exp\left((g_1-\chi_2)\frac{z}{2}\right)$$

The exact transfer matrix of: Im-only-detuning & perfect-PT case

$$\frac{\partial}{\partial z}\begin{pmatrix}A_1\\A_2\end{pmatrix}=i\begin{pmatrix}\beta_0+ig_1 & \kappa\\ \kappa & \beta_0-ig_1\end{pmatrix}\begin{pmatrix}A_1\\A_2\end{pmatrix}$$

$\Delta_{\mathrm{im}}=g_1$ Im detuning

0 if PT-sym !!

$\delta=+i\Delta_{\mathrm{im}}=ig_1$ complex detuning

below EP

$\Omega=\sqrt{\kappa^2-g_1^2}$ detuning & coupling

$\begin{pmatrix}A_1(z)\\A_2(z)\end{pmatrix}=M(z)\begin{pmatrix}A_1(0)\\A_2(0)\end{pmatrix}$ Transfer matrix

$$M(z)=\begin{pmatrix}\cos(\Omega z)+\dfrac{g_1}{\sqrt{\kappa^2-g_1^2}}\sin(\Omega z) & \dfrac{i\kappa}{\sqrt{\kappa^2-g_1^2}}\sin(\Omega z)\\ \dfrac{i\kappa}{\sqrt{\kappa^2-g_1^2}}\sin(\Omega z) & \cos(\Omega z)-\dfrac{g_1}{\sqrt{\kappa^2-g_1^2}}\sin(\Omega z)\end{pmatrix}\exp\left(0\frac{z}{2}\right)$$

The exact transfer matrix of no-detuning perfect case

$$\frac{\partial}{\partial z}\begin{pmatrix}A_1\\A_2\end{pmatrix} = i\begin{pmatrix}\beta_0 + i0 & \kappa\\ \kappa & \beta_0 - i0\end{pmatrix}\begin{pmatrix}A_1\\A_2\end{pmatrix}$$

$\Delta_{\text{im}} = 0$ Im detuning

$\delta = 0$ complex detuning

$$\begin{pmatrix}A_1(z)\\A_2(z)\end{pmatrix} = M(z)\begin{pmatrix}A_1(0)\\A_2(0)\end{pmatrix}$$ Transfer matrix

$\Omega = \kappa$

$$M(z) = \begin{pmatrix}\cos(\kappa z) & i\sin(\kappa z)\\ i\sin(\kappa z) & \cos(\kappa z)\end{pmatrix}$$ Simply rotation

Switching requirements

⊗ cross

$\Delta_{\text{im}} = g_1$

$$\Omega = \sqrt{\kappa^2 - g_1^2}$$

$$M(z) = \begin{pmatrix}\cos(\Omega z) + \frac{g_1}{\sqrt{\kappa^2 - g_1^2}}\sin(\Omega z) & \frac{i\kappa}{\sqrt{\kappa^2 - g_1^2}}\sin(\Omega z)\\ \frac{i\kappa}{\sqrt{\kappa^2 - g_1^2}}\sin(\Omega z) & \cos(\Omega z) - \frac{g_1}{\sqrt{\kappa^2 - g_1^2}}\sin(\Omega z)\end{pmatrix}\exp(0\frac{z}{2})$$

0 1? 1 0

etc.

⊖ bar

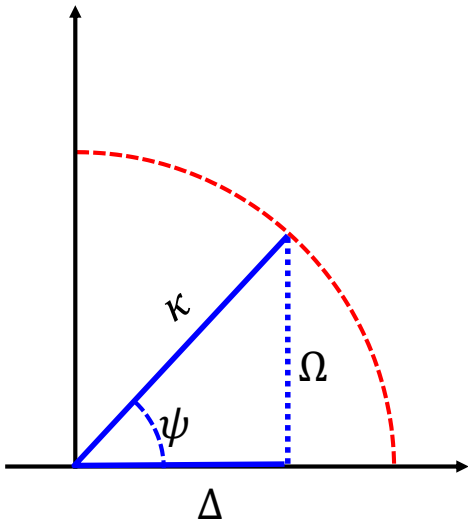
$\Delta_{\text{im}} = 0$

$$M(z) = \begin{pmatrix}\cos(\kappa z) & i\sin(\kappa z)\\ i\sin(\kappa z) & \cos(\kappa z)\end{pmatrix}$$

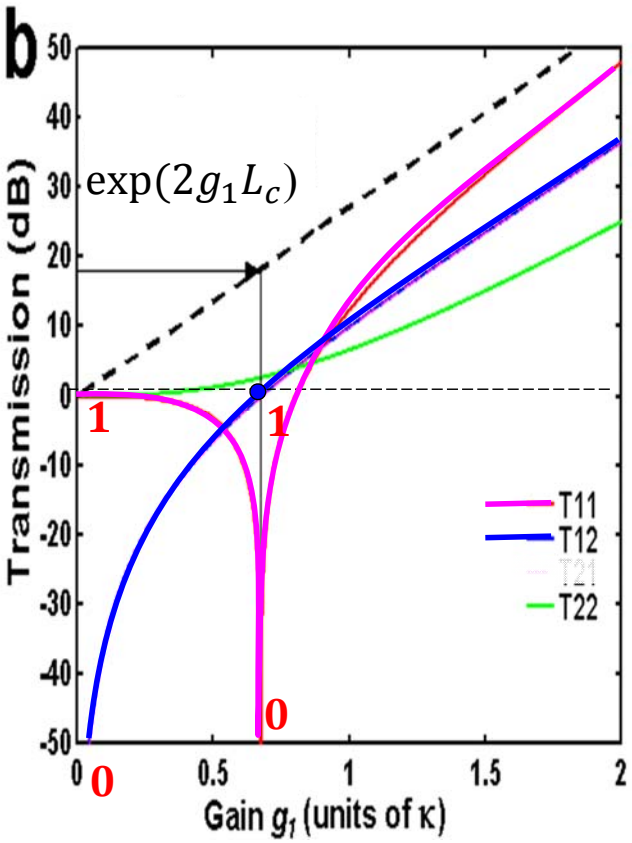
1 0

→ Change of beat length
→ cos & sin cancellations

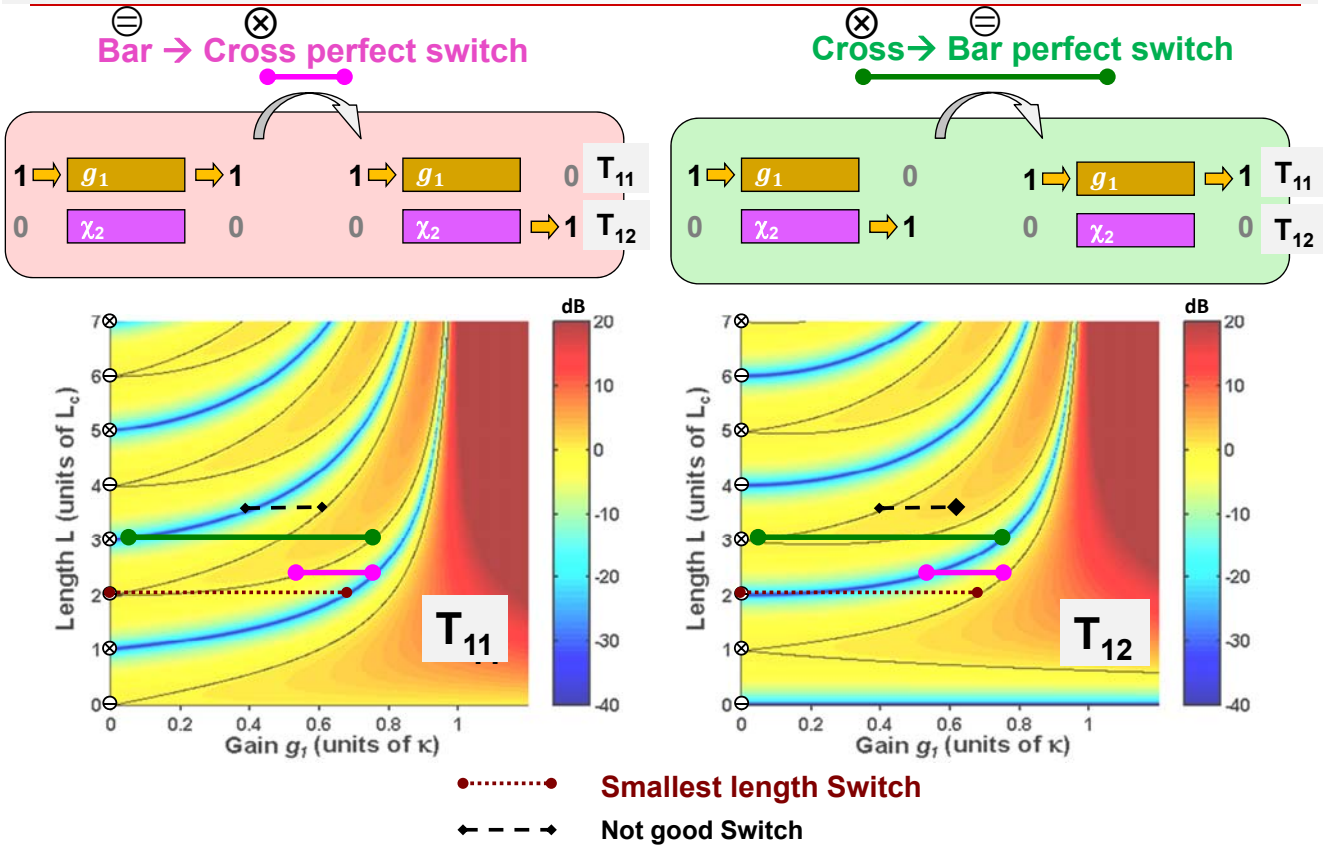
Switching and all the transfer matrix



$$\pi \sin\psi = \pi - \psi$$



Bar and Cross perfect switch states in ideal (gain=loss) PT-symmetric coupler (PTSC)



What is the new concept replacing "V_π" ?

Real index modulation → **Imaginary index modulation**

→ **Δ(Im) replaces Δ(Re)**

→ **Δ(Im)=|*g*|+|χ| (the « sum » of gain and loss)**

CMT solution → $\tan\left(\sqrt{\pi^2-\Delta_{\text{im}}^2 L^2}\right)=-\sqrt{\frac{\pi^2}{\Delta_{\text{im}}^2 L^2}-1}$

$$\Delta_{\text{im}} L = 0.67\pi = 2.103$$

$< 1.00\ \pi$

(usual electro-optic switch's $\Delta\beta L$ product)

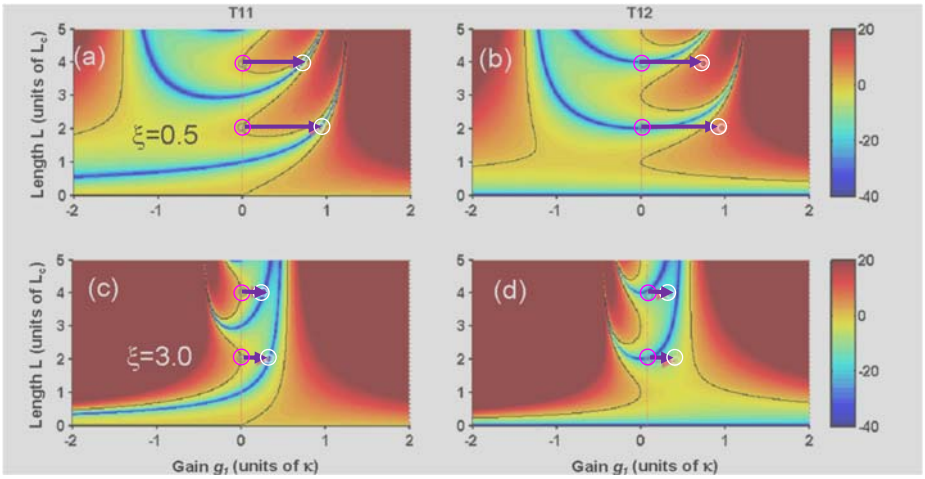
So "33% shorter" couplers are possible

(insofar as measured by phase)

What about less perfect systems?

$\chi_2 = \xi g_1$

$\xi > 1$ or $\xi < 1$



« 0 » always exist
« 1 » level is higher
or lower than unity
But if ±5 dB are tolerable
Then large ξ margin exist !

Extra dB in T12



References on switching and coupled guides/plasmonics

[1] H. Benisty, A. Degiron, A. Lupu, A. De Lustrac, S. Chénais, S. Forget, M. Besbes, G. Barbillon, A. Bruyant, S. Blaize, and G. Lérondel**, "Implementation of PT symmetric devices using plasmonics: principle and applications,"** Optics Express, vol. 19, pp. 18004-18019, 2011.

[2] H. Benisty, C. Yan, A. T. Lupu, and A. Degiron, "**Healing Near-PT-Symmetric Structures to Restore Their Characteristic Singularities: Analysis and Examples,**" IEEE J. Lightwave Technol., vol. 30, pp. 2675-2683, 2012.

[3] A. Lupu, H. Benisty, and A. Degiron, **"Switching using PT symmetry in plasmonic systems: positive role of the losses**," Opt. Express, vol. 21, pp. 21651-21668, 6 2013.

[4] A. Lupu, H. Benisty, and A. Degiron, "**Using optical PT-symmetry for switching applications,**" Photonics and Nanostructures-Fundamentals and Applications, vol. 12, pp. 305-311, 2014.

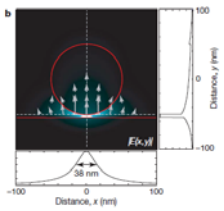
[5] H. Benisty, A. Lupu, and A. Degiron, "**Transverse periodic PT symmetry for modal demultiplexing in optical waveguides**," Phys. Rev. A, vol. 91, p. 053825, 2015.

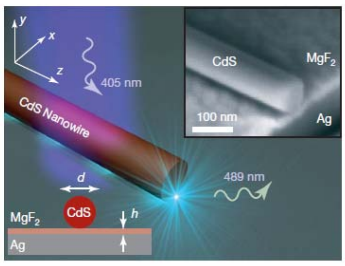
[6] H. Benisty and M. Besbes, "Plasmonic inverse rib waveguiding for tight confinement and smooth interface definition " J. Appl. Phys., vol. 108, pp. 063108 (1-8), 2010.

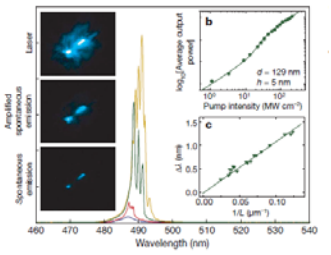
[7] H. Benisty and M. Besbes, "Confinement and optical properties of the plasmonic inverse-rib waveguide," J. Opt. Soc. Am. B, vol. 29, pp. 818-826, March 29 2012.

CONTEXT : Gain with plasmons

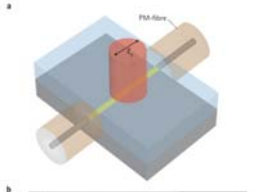
- **SPASER (Stockman, Oulton with nanorods, …)**

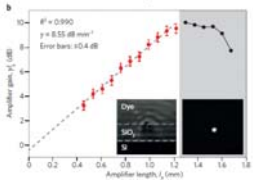


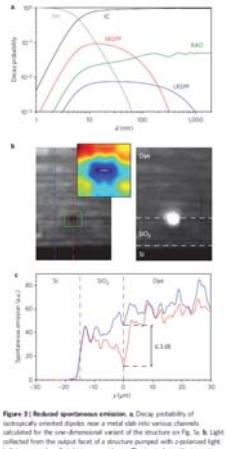




- **Optical Amplifiers with LRSPP (Berini)**

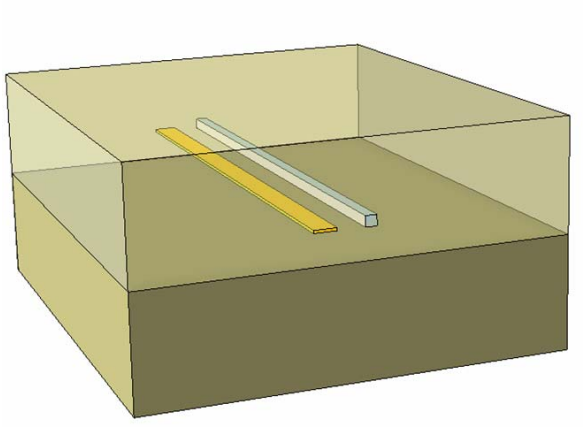


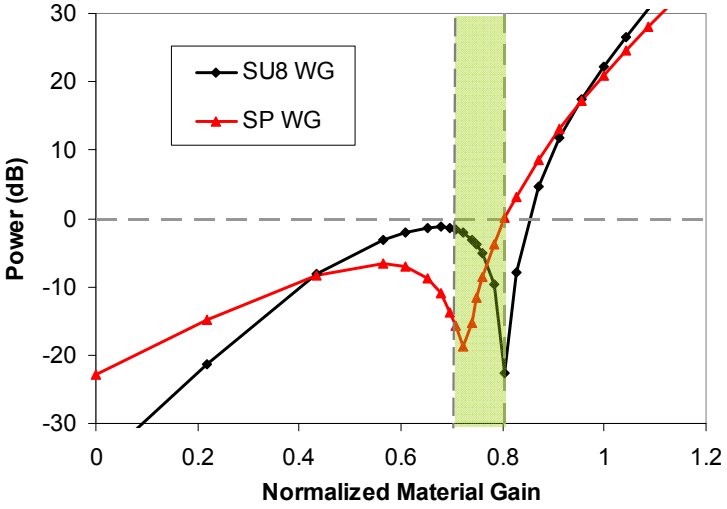




Application example : Plasmonic modulator

- No électro-optic modulation in metals
- $\mathcal{PT}$-symmetry offers a plausible alternative to elaborate active devices.

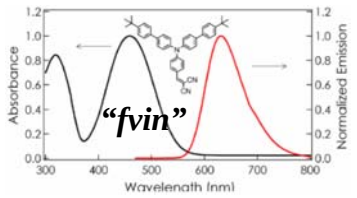




Lupu et al., Opt. Express 2013

Model Realisation : Gain with organics

S. Chénais & S. Forget team, LPL, Paris 13

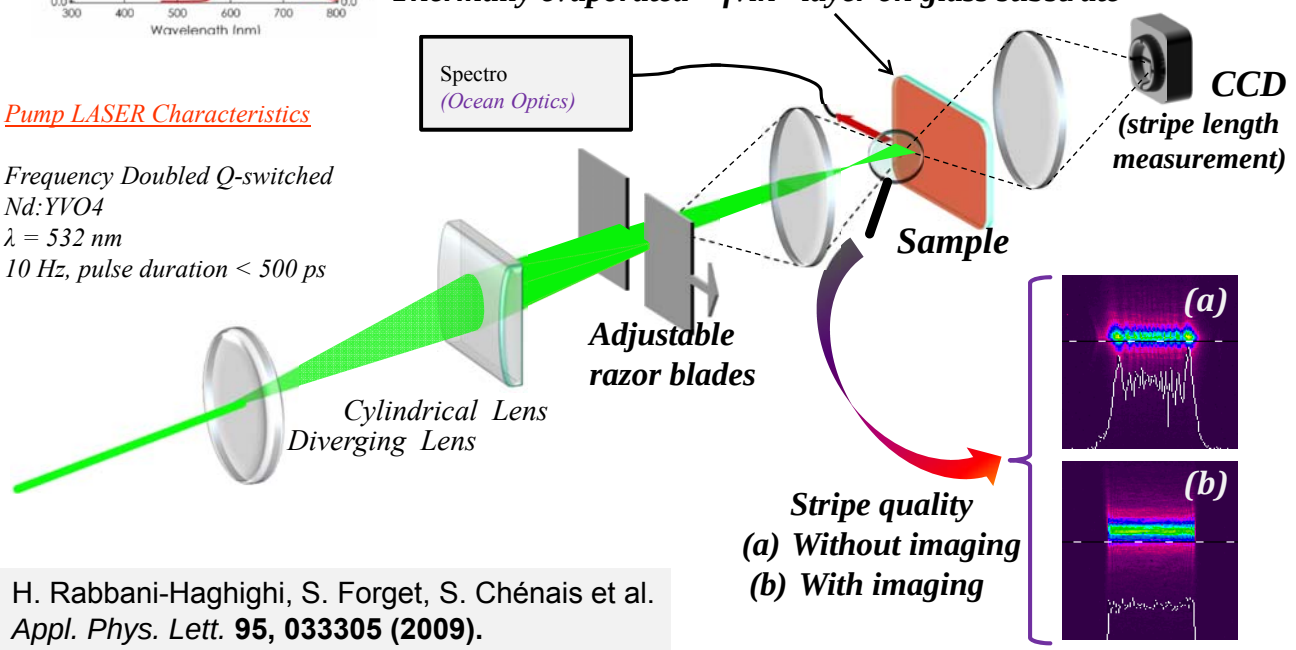


Variable Stripe Length (VSL) technique:
+ Molecular film

Thermally evaporated "fvin" layer on glass substrate

Pump LASER Characteristics

Frequency Doubled Q-switched
Nd:YVO4
λ = 532 nm
10 Hz, pulse duration < 500 ps

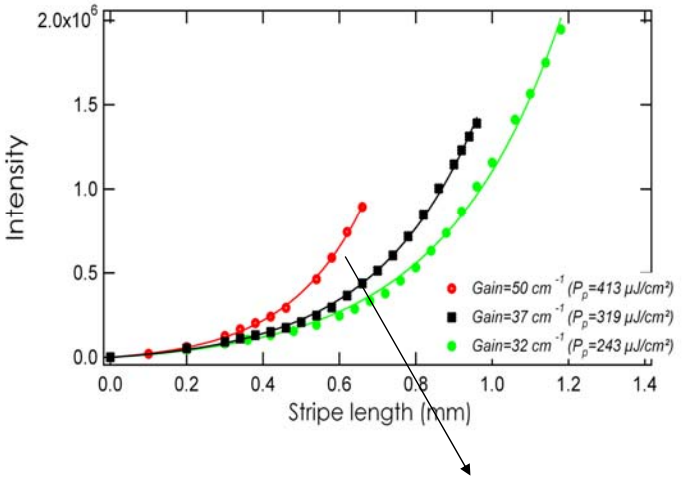


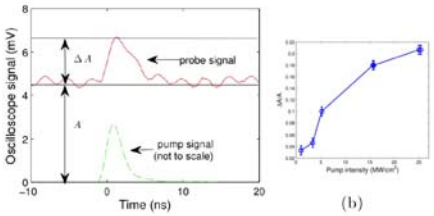
H. Rabbani-Haghighi, S. Forget, S. Chénais et al. *Appl. Phys. Lett.* **95, 033305 (2009).**

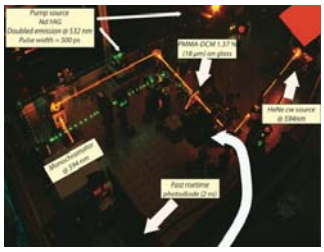
Pump Stripe Width : 320 µm

Gain with organics can be high & fast

S. Chénais & S. Forget team, LPL, Paris 13







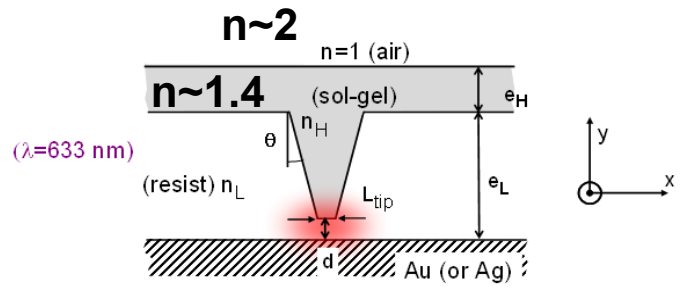
E=0,41 mJ/cm² → Gain= 50 cm⁻¹

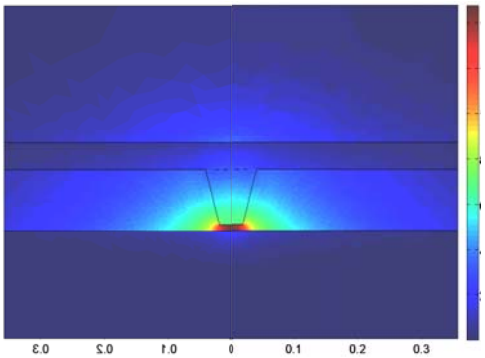
Comparison : in a 5%-doped DCM:PMMA of same thickness :

Gain= 40 cm⁻¹

nanosec. pump-probe gain

More miniature plasmonics ? Example of Plasmonic Inverse-Rib Optical WG



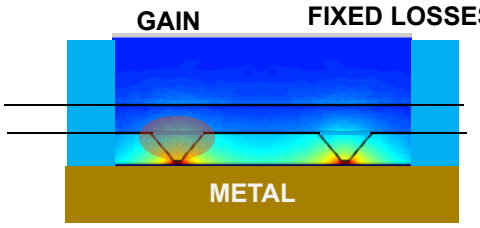




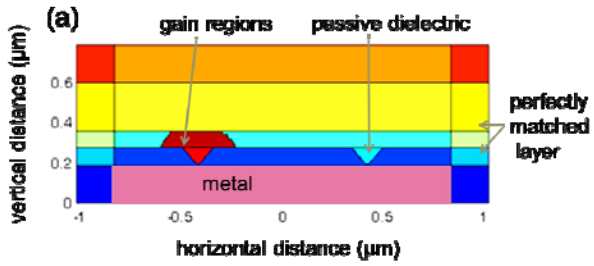
- **JAP 2010, H. Benisty and M. Besbes E-field in 30-50 nm tip... (+JOSA 2012)**
- **Like Oulton's nanorod/spaser, but deterministic**
- **Single WG** ? Gain brings just … loss compensation …Then yeah gain !

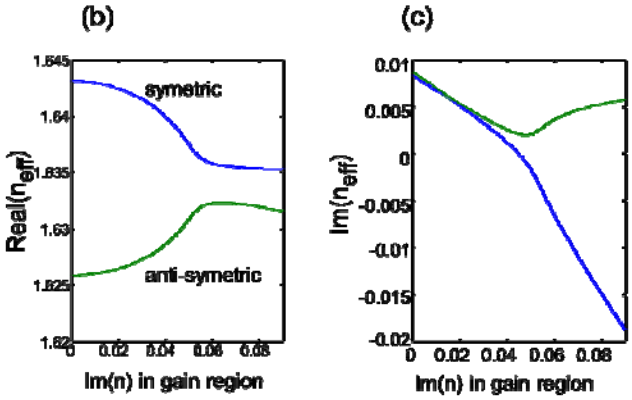
Two coupled «PIROWs » : good ?

A gain one and a normal one



Can we have a good EP ?





H. Benisty and M. Besbes,

"Confinement and optical properties of the plasmonic inverse-rib waveguide,"

JOSA. B, vol. 29, pp. 818-826, 2012.

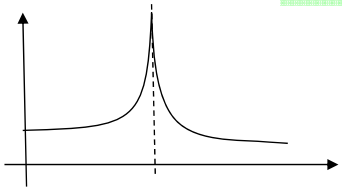
Something wrong ?

« Danger, LASER !»

- Fabry-Perot formula with gain

$$A_{\text{out}} = A_{in}\left(\frac{t^2}{1 - r^2\, e^{+2g_{\text{amp}}L_{\text{cav}}} e^{2ikL_{\text{cav}}}}\right)$$

- Pole at $r\, e^{+g_{\text{amp}}L_{\text{cav}}}=1$



→ Well-known **Threshold for lasing**
(no amplification limit within a linear saturation-free ansatz)

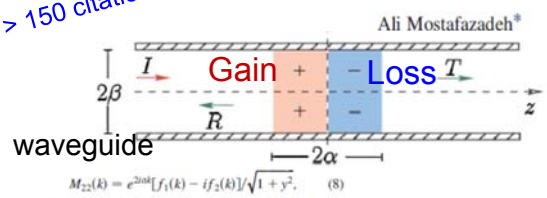
• But …

PRL 102, 220402 (2009) PHYSICAL REVIEW LETTERS

Spectral Singularities of Complex Scattering Potentials and Infinite Reflection and Transmission Coefficients at Real Energies

> 150 citations

Ali Mostafazadeh*



$$M_{22}(k) = e^{2ika}[f_1(k) - if_2(k)]/\sqrt{1+y^2}, \quad (8)$$

where f_1 and f_2 are real-valued functions given by

$$f_1(k) = \sqrt{1+y^2}\,|\cos(akw)|^2 - |\sin(akw)|^2, \quad (9)$$

$$f_2(k) = \mathrm{Re}[\sqrt{1+iy}(2-iy)\sin(akw)\cos(akw^*)], \quad (10)$$

$y := \beta/k^2$, $w := \sqrt{1-iy}$, and Re means "real part of."

$$\tan(akw) = -\left[\frac{\sqrt{1-iy}(2+iy)}{\sqrt{1+iy}(2-iy)}\right]\tan(akw)^*. \quad (12)$$

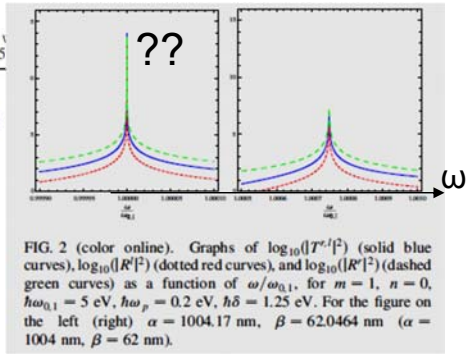


FIG. 2 (color online). Graphs of $\log_{10}(|T^{r,l}|^2)$ (solid blue curves), $\log_{10}(|R^l|^2)$ (dotted red curves), and $\log_{10}(|R^r|^2)$ (dashed green curves) as a function of $\omega/\omega_{0,1}$, for $m = 1$, $n = 0$, $\hbar\omega_{0,1} = 5$ eV, $\hbar\omega_p = 0.2$ eV, $\hbar\delta = 1.25$ eV. For the figure on the left (right) $\alpha = 1004.17$ nm, $\beta = 62.0464$ nm ($\alpha = 1004$ nm, $\beta = 62$ nm).

No mention of « lasing » or « threshold » ??

→ Actually a sign that PT-symmetry needs our work to be a melting-pot !

CONCLUSION
Basic (transverse) PT-symmetry

→ New use of gain beyond plain loss compensation

→ New entry for open systems (new inner products…) thanks to new quantum theory input

→ OK for Adaptation to plasmonics (constant loss media)

References on switching and coupled guides/plasmonics

[1] H. Benisty, A. Degiron, A. Lupu, A. De Lustrac, S. Chénais, S. Forget, M. Besbes, G. Barbillon, A. Bruyant, S. Blaize, and G. Lérondel**, "Implementation of PT symmetric devices using plasmonics: principle and applications,"** Optics Express, vol. 19, pp. 18004-18019, 2011.

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[3] A. Lupu, H. Benisty, and A. Degiron, **"Switching using PT symmetry in plasmonic systems: positive role of the losses**," Opt. Express, vol. 21, pp. 21651-21668, 6 2013.

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[7] H. Benisty and M. Besbes, "Confinement and optical properties of the plasmonic inverse-rib waveguide," J. Opt. Soc. Am. B, vol. 29, pp. 818-826, March 29 2012.