Outline of Part 2

Metamaterial to get real-space pseudo magnetic field

- **High level description and design strategy of devices (~ TO)**
- **Respecting T symmetry – analogy to spin Hall effect of light**

- Gauge transformation & associated metamaterials
  - Effective gauge field
  - Splitting the center of local dispersion surface here
- Applications
  - Pure gauge field – angle-independent polarization control
  - Non-trivial gauge field – Pseudo magnetic field

    Optical spin-Hall effect

    One-way edge state

    Invisibility cloaking

Gauge Transformation in QM

- Schrodinger equation

$$\frac{1}{2m}(-i\hbar\nabla + eA)^2\psi - e\Phi\psi = i\hbar\partial_t\psi$$

- Gauge transformation
  - On the wave function

$$\psi' = \exp\left(-i\frac{e}{\hbar}f\right)\psi$$

  - On the potentials

$$A' = A + \nabla f$$
$$\Phi' = \Phi - \partial_t f$$

Gauge transformation

- Special class of field transformation (FT) for waves on x-y plane

$$\begin{pmatrix} E_z' \\ iH_z' \end{pmatrix} = \begin{pmatrix} \cos k_0\phi & -\sin k_0\phi \\ \sin k_0\phi & \cos k_0\phi \end{pmatrix} \begin{pmatrix} E_z \\ iH_z \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{E}_T' \\ i\boldsymbol{H}_T' \end{pmatrix} = \begin{pmatrix} \cos k_0\phi & \sin k_0\phi \\ -\sin k_0\phi & \cos k_0\phi \end{pmatrix} \begin{pmatrix} \boldsymbol{E}_T \\ i\boldsymbol{H}_T \end{pmatrix}$$

⇒Reciprocal medium only

Phys. Rev. Lett. 111, 033901 (2013)
General: Tretyakov, NJP 10, 115028 (2008)

- Gauge field medium = TO medium + tilted anisotropy A_x, A_y

$$\bar{\bar{\epsilon}} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & A_y \\ \epsilon_{xy} & \epsilon_{yy} & -A_x \\ A_y & -A_x & \epsilon_{zz} \end{pmatrix} \qquad \bar{\bar{\mu}} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & -A_y \\ \epsilon_{xy} & \epsilon_{yy} & A_x \\ -A_y & A_x & \epsilon_{zz} \end{pmatrix}$$

Gauge Transformation

$$\bar{\bar{\epsilon}}' = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & A'_y \\ \epsilon_{xy} & \epsilon_{yy} & -A'_x \\ A'_y & -A'_x & \epsilon_{zz} \end{pmatrix} \qquad \bar{\bar{\mu}}' = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & -A'_y \\ \epsilon_{xy} & \epsilon_{yy} & A'_x \\ -A'_y & A'_x & \epsilon_{zz} \end{pmatrix}$$

Pseudo Vector Potential : $\boldsymbol{A}' = \boldsymbol{A} + \nabla\phi$
Pseudo Magnetic Field: $\boldsymbol{B}_{\boldsymbol{eff}} = \nabla \times \boldsymbol{A}$

$$\boldsymbol{A} = A_x\hat{x} + A_y\hat{y}$$

Phys. Rev. Lett. 114, 103902 (2015)

Details on identifying the gauge field

$$\begin{pmatrix} E_z' \\ iH_z' \end{pmatrix} = \begin{pmatrix} \cos k_0\phi & -\sin k_0\phi \\ \sin k_0\phi & \cos k_0\phi \end{pmatrix} \begin{pmatrix} E_z \\ iH_z \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{E}_T' \\ i\boldsymbol{H}_T' \end{pmatrix} = \begin{pmatrix} \cos k_0\phi & \sin k_0\phi \\ -\sin k_0\phi & \cos k_0\phi \end{pmatrix} \begin{pmatrix} \boldsymbol{E}_T \\ i\boldsymbol{H}_T \end{pmatrix}$$

Induces a corresponding transformation of the "potentials"

- Global symmetry (constant ϕ leaves the material invariant)

$$\bar{\bar{\epsilon}}^{(0)} = \bar{\bar{\mu}}^{(0)} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{xy} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix}$$

- Promoting the global symmetry to a local one $\phi(x, y)$

$$\bar{\bar{\epsilon}} \to \bar{\bar{\epsilon}} + \Delta\bar{\bar{\epsilon}}, \qquad \bar{\bar{\mu}} \to \bar{\bar{\mu}} + \Delta\bar{\bar{\mu}}$$

$$\Delta\bar{\bar{\epsilon}} = -\Delta\bar{\bar{\mu}} = \begin{pmatrix} 0 & 0 & \partial_y\phi \\ 0 & 0 & -\partial_x\phi \\ \partial_y\phi & -\partial_x\phi & 0 \end{pmatrix}$$

Details on identifying the gauge field

- Finally, consder the form

$$\bar{\bar{\epsilon}} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & A_y \\ \epsilon_{xy} & \epsilon_{yy} & -A_x \\ A_y & -A_x & \epsilon_{zz} \end{pmatrix}, \qquad \bar{\bar{\mu}} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & -A_y \\ \epsilon_{xy} & \epsilon_{yy} & A_x \\ -A_y & A_x & \epsilon_{zz} \end{pmatrix}$$

- Gauge transformtion gives

$$\boldsymbol{A} \to \boldsymbol{A} + \nabla\phi$$

- Wave equation

$$(\nabla \pm ik_0\boldsymbol{A}) \cdot \frac{1}{m} \cdot (\nabla \pm ik_0\boldsymbol{A})\psi + k_0^2\epsilon_{zz}\psi = 0 \qquad m = \begin{pmatrix} \epsilon_{yy} & -\epsilon_{xy} \\ -\epsilon_{xy} & \epsilon_{xx} \end{pmatrix}$$

Metamaterial realization

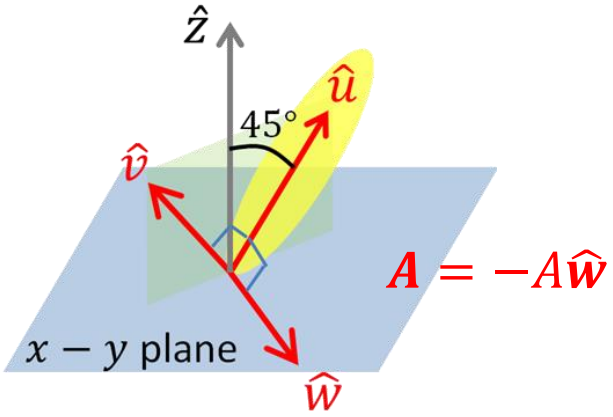
- We consider simple material with $A_x = 0$ and $A_y \neq 0$

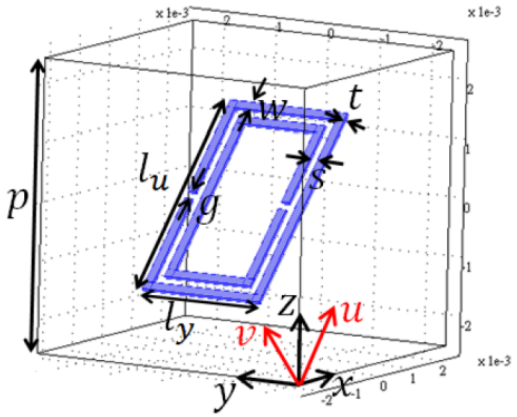
$$\bar{\bar{\epsilon}} = \begin{pmatrix} n & 0 & A_y \\ 0 & n & 0 \\ A_y & 0 & n \end{pmatrix}, \bar{\bar{\mu}} = \begin{pmatrix} n & 0 & -A_y \\ 0 & n & 0 \\ -A_y & 0 & n \end{pmatrix}$$

- Rotate by $45°$ about y:

$$\epsilon_u = \mu_v = n + A_y, \epsilon_v = \mu_u = n - A_y, \epsilon_y = \mu_y = n$$

- It is anisotropic and can be realized by metamaterials



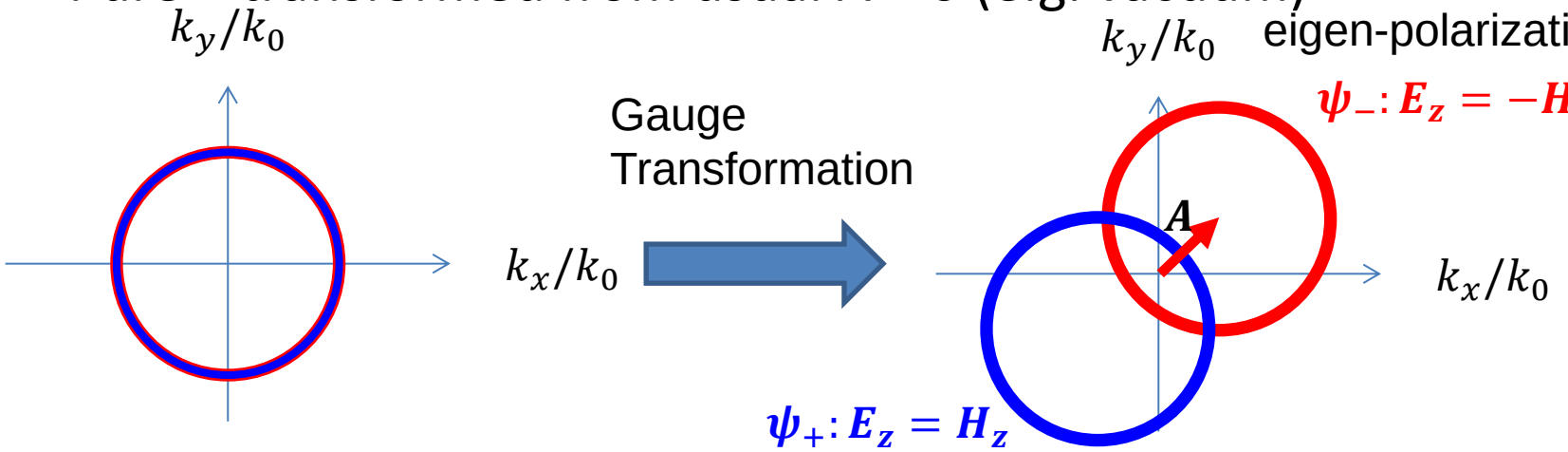


Gauge-field metamaterials

- Rotates about z-direction gives A_x

An example of Pure Gauge field

- Splitting of dispersion surface into pseudo spin-up/down
- Pure = transformed from usual A = 0 (e.g. vacuum)



Vacuum

$$\boldsymbol{A} = 0$$

$$\bar{\bar{\epsilon}} = \bar{\bar{\mu}} = I$$

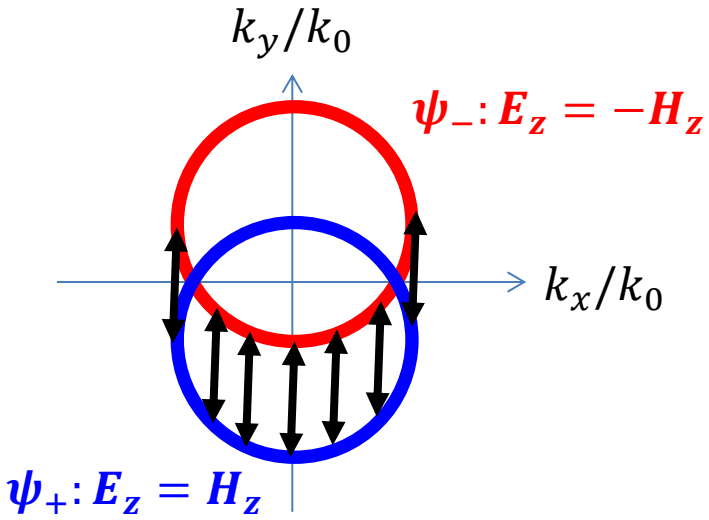
Gauge-field medium

$$\boldsymbol{A} = \nabla\phi$$

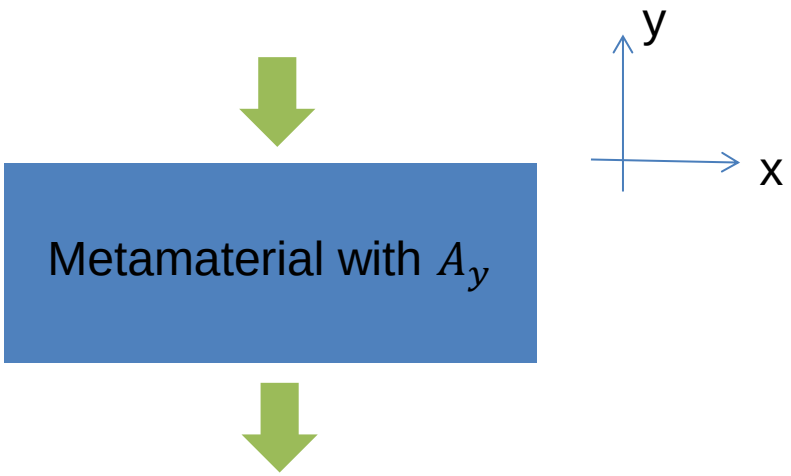
$$\bar{\bar{\epsilon}} - I = I - \bar{\bar{\mu}} = \begin{pmatrix} 0 & 0 & A_y \\ 0 & 0 & -A_x \\ A_y & -A_x & 0 \end{pmatrix}$$

Angle-independent polarization control

- Polarization control = phase difference between 2 eigen polarizations

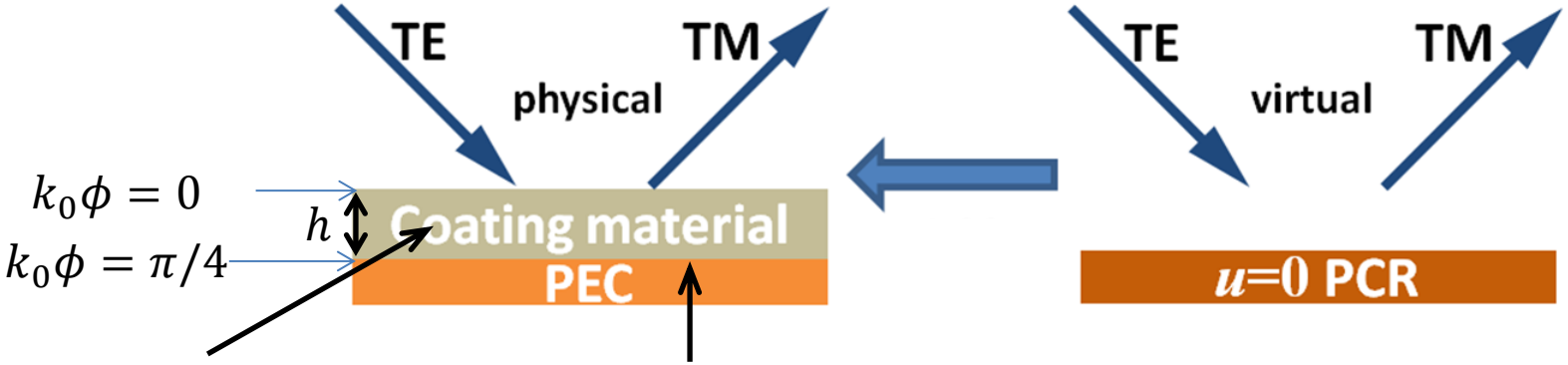


Constant splitting for any incident angles



All-angle polarization control

$$\begin{pmatrix} E_z' \\ iH_z' \end{pmatrix} = \begin{pmatrix} \cos k_0\phi & -\sin k_0\phi \\ \sin k_0\phi & \cos k_0\phi \end{pmatrix} \begin{pmatrix} E_z \\ iH_z \end{pmatrix}$$



Gauge-field metamaterial

$$\epsilon_{xz} = \epsilon_{zx} = -\frac{\pi}{4h} \qquad E_z' = E_x' = 0$$

$$\mu_{xz} = \mu_{zx} = \frac{\pi}{4h}$$

Gauge medium with $A_y = -\frac{\pi}{4h}$

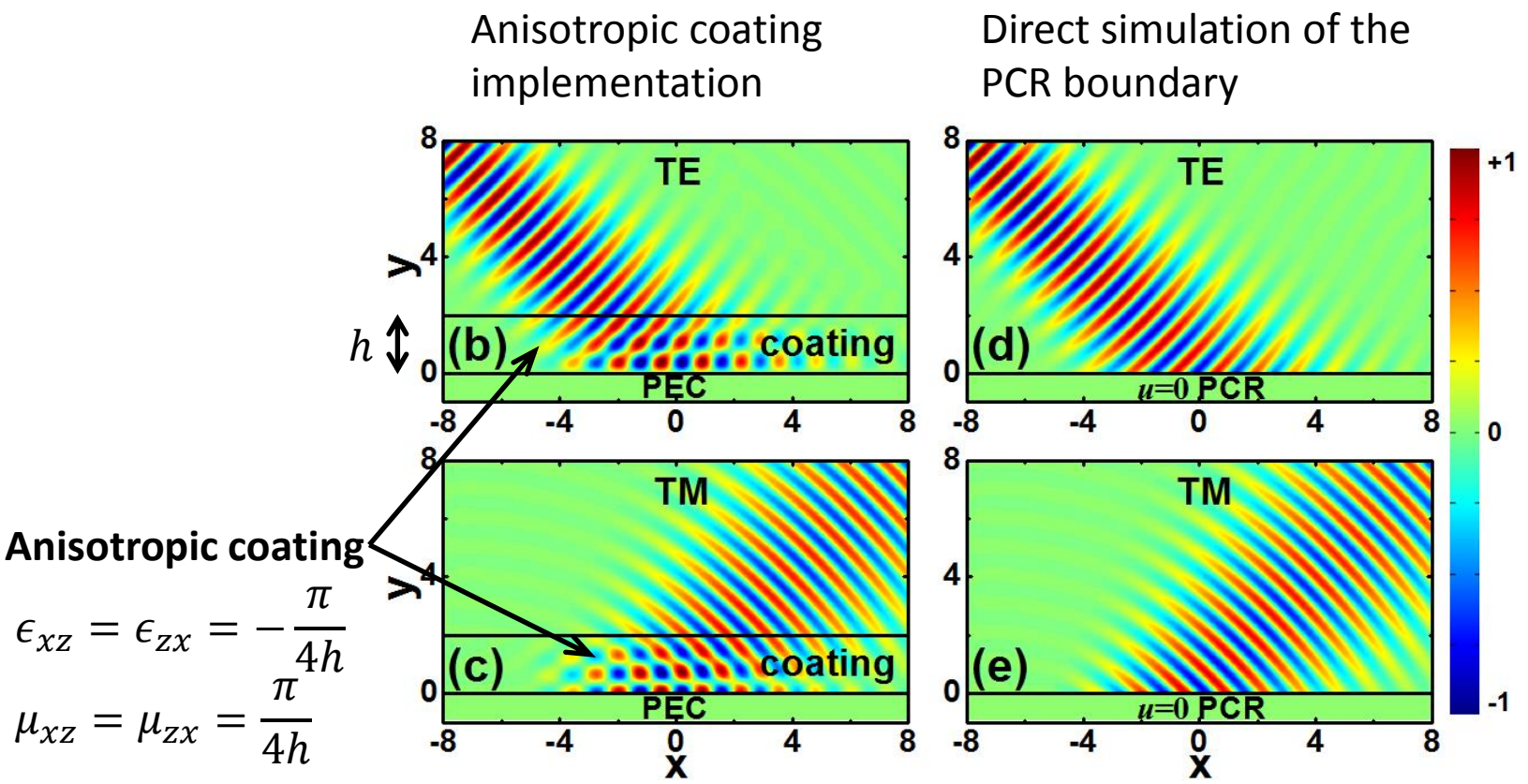
Equivalent artificial boundary condition

$$E_z - iH_z = 0,$$
$$E_x + iH_x = \partial_y(E_z + iH_z) = 0$$

to give TE/TM conversion on reflection

All-angle polarization control

- Complete cross-LP conversion



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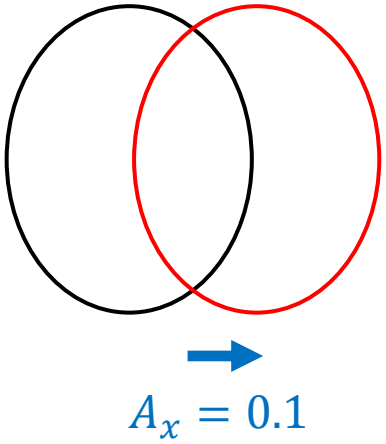
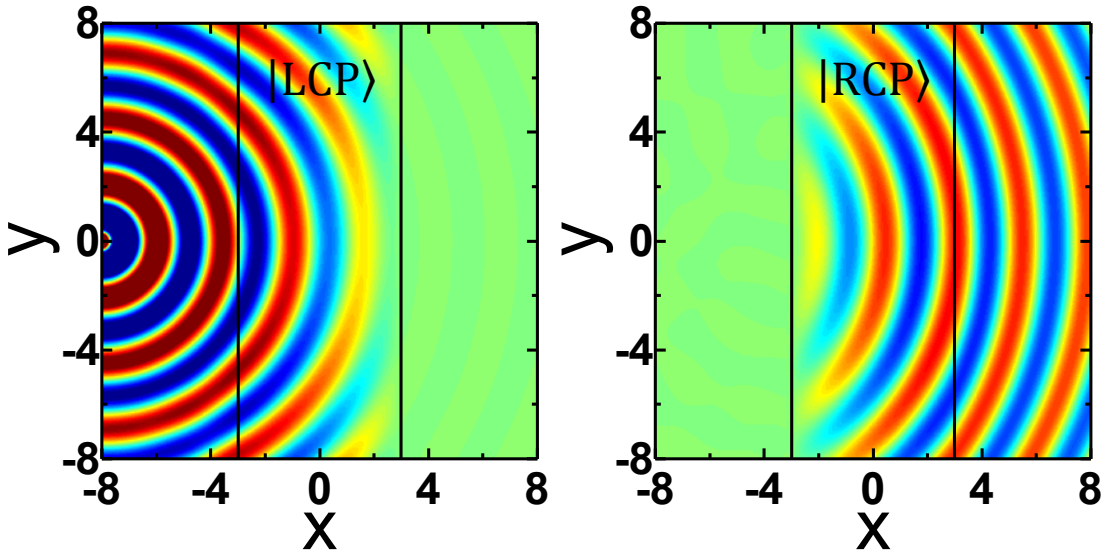
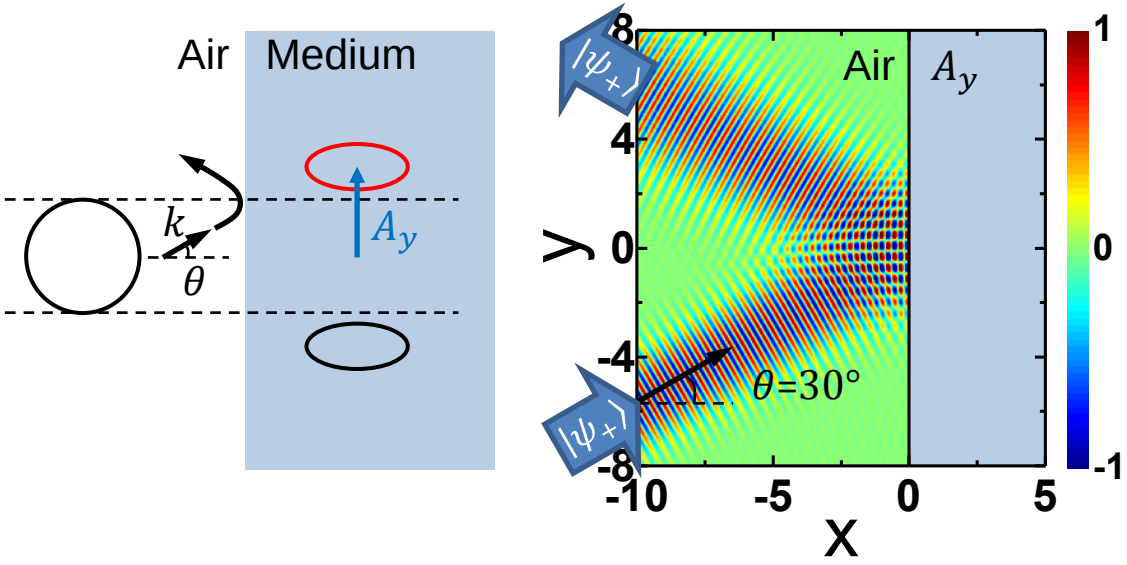
All-angle polarization control

- Equivalent argument on circular polarization
- Constant gauge field A_x





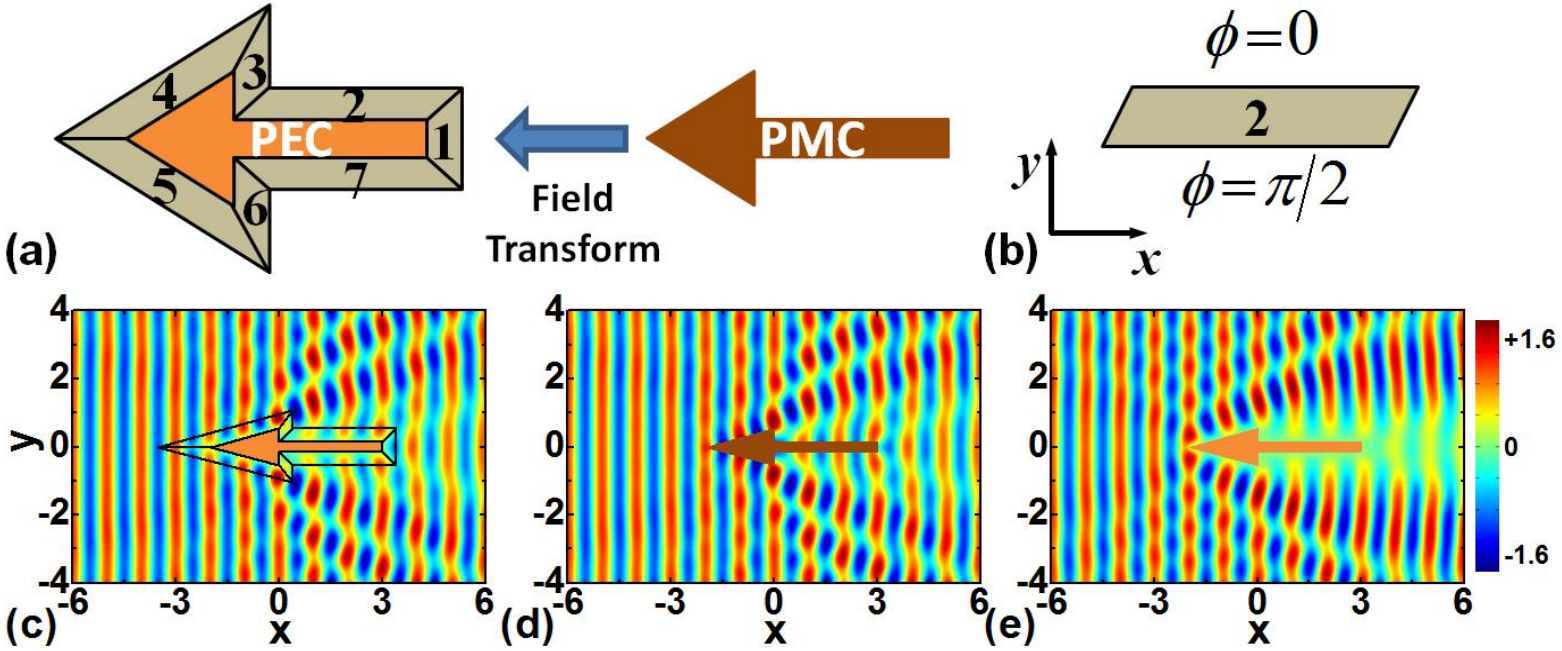
Co-spin mirror



Mimicking PMC

- Similarly, we can also mimicking a PMC object with field transformation.

$$\begin{pmatrix} E_z' \\ iH_z' \end{pmatrix} = \begin{pmatrix} \cos k_0\phi & -\sin k_0\phi \\ \sin k_0\phi & \cos k_0\phi \end{pmatrix} \begin{pmatrix} E_z \\ iH_z \end{pmatrix}$$



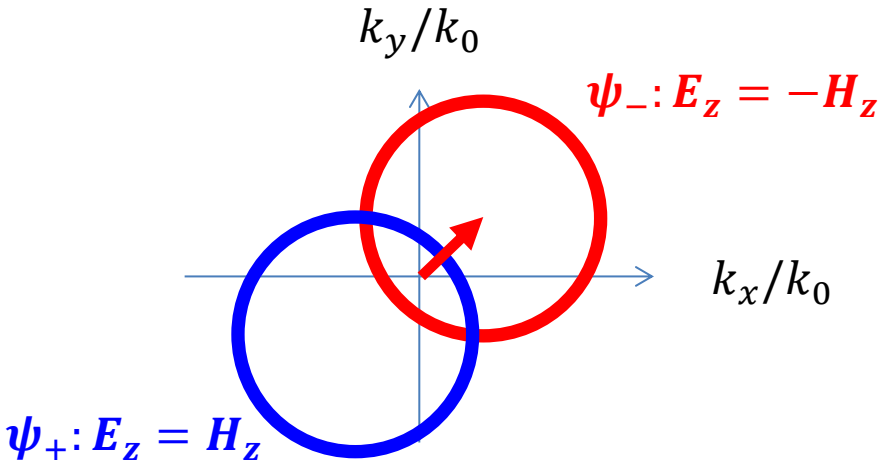
PEC object with anisotropic coating

Bare PMC object

Bare PEC object

Non-trivial Gauge

- No transformation can bring it back to a medium of $\boldsymbol{A} = 0$
- The generation of pseudo-magnetic field
  - Not a real magnetic field, mimicked by anisotropy of metamaterials



Gauge-field medium

$$\boldsymbol{A} = \nabla\phi$$

$$\boldsymbol{B_{eff}} = \nabla \times \boldsymbol{A} = B\hat{z}$$

- Spatially varying $\boldsymbol{A}$
- Non-zero $\boldsymbol{B_{eff}}$

Geometrical optics

- Wave equation

$$(\nabla \pm ik_0\boldsymbol{A}) \cdot \frac{1}{n}(\nabla \pm ik_0\boldsymbol{A})\psi_\pm + k_0^2 n\psi_\pm = 0$$

- Local wave vector

$$\boldsymbol{k} = \nabla \arg \psi_\pm$$

- Classical Hamiltonian by $\nabla \to i\boldsymbol{k}$

$$\mathcal{H}(\boldsymbol{k}, \boldsymbol{r}) = \frac{1}{2k_0 n}(|\boldsymbol{k} \pm k_0\boldsymbol{A}|^2 - k_0^2 n^2) = 0$$

- Hamiltonian dynamics

$$\frac{d\boldsymbol{r}}{ds} = \frac{\partial \mathcal{H}}{\partial \boldsymbol{k}} = \frac{\boldsymbol{k} \pm k_0\boldsymbol{A}}{k_0 n}$$

$$\frac{d\boldsymbol{k}}{ds} = -\frac{\partial \mathcal{H}}{\partial \boldsymbol{r}} = \frac{|\boldsymbol{k} \pm k_0\boldsymbol{A}|^2 + k_0^2 n^2}{2k_0 n^2}\nabla n \mp k_0(\nabla \boldsymbol{A}) \cdot \frac{d\boldsymbol{r}}{ds}$$

Optical spin-Hall effect

- In geometrical optics limit, ray equation becomes:

~E field ~B field

$$\frac{d}{ds}\left(n\frac{d\boldsymbol{r}}{ds}\right) = \nabla n \mp \hat{s}\times(\nabla\times\boldsymbol{A})$$

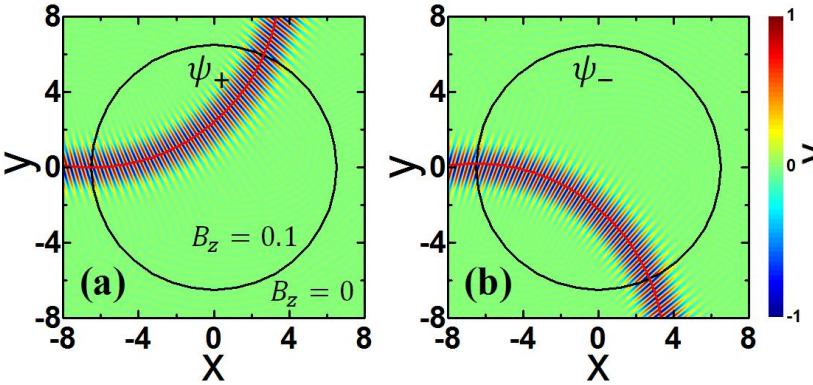
Previous ray equation

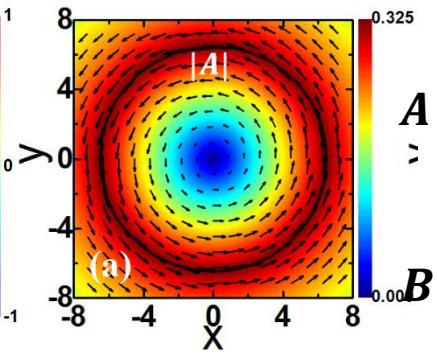
Lorentz force for photon

$\mp: E_z = \pm H_z$

s: arc length of the ray
$\boldsymbol{r}$: position vector
$\hat{s}$: propagating direction

- In addition to pseudo-electric force (∇n), the "magnetic force" for photon ($\mp\hat{s}\times\boldsymbol{B_{eff}}$) provides optical spin Hall effect





$$\boldsymbol{A} = \begin{cases} 1/2B_z r\hat{\phi} & r \le r_0 \\ 1/2B_z r_0^2/r\hat{\phi} & r > r_0 \end{cases}$$

$$\boldsymbol{B} = \nabla\times\boldsymbol{A} = \begin{cases} B_z\hat{z} & r \le r_0 \\ 0 & r > r_0 \end{cases}$$

Phys. Rev. Lett. 114, 103902 (2015)

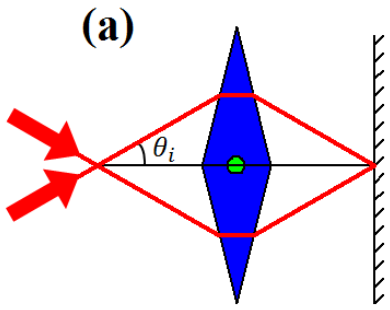
Photonic Aharonov-Bohm effect

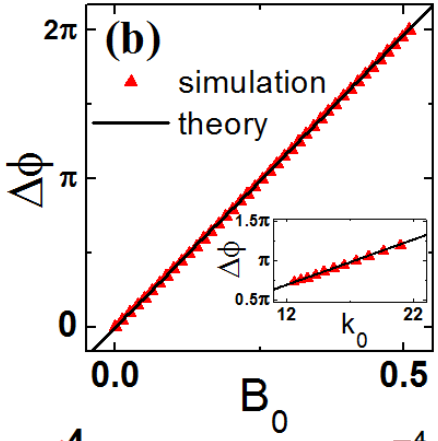
F. Liu, and Jensen Li, *Phys. Rev. Lett.* **114**, 103902(2015)

- An additional path-dependent geometric phase exists for a non-trivial gauge field profile (Aharonov-Bohm effect).
- Non-zero round-trip propagation phase:

$$\phi_g = \mp \oint_{\partial\Sigma} k_0 \boldsymbol{A} \cdot d\boldsymbol{r} = \mp k_0 \iint_{\Sigma} \boldsymbol{B}_{\text{eff}} \cdot d\boldsymbol{\Sigma}$$

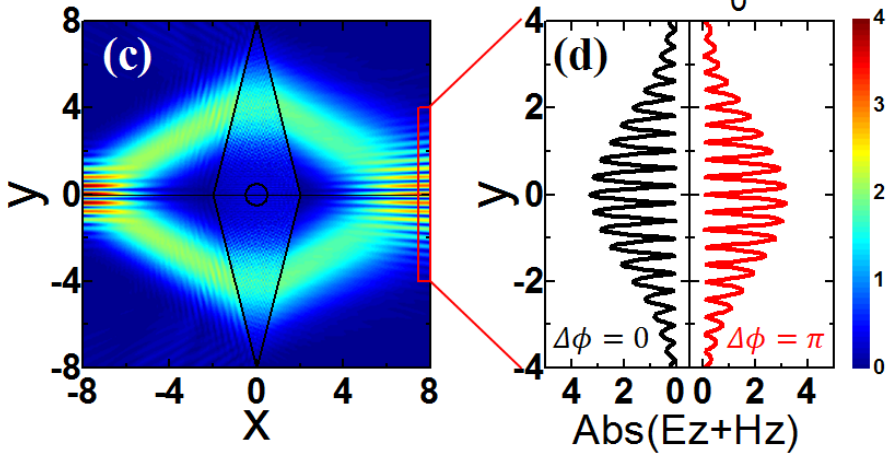
- Zero $\boldsymbol{B}_{\text{eff}}$ on the paths (gauge field exists)
- Two ψ_+ beams interference on screen





- Closed loop phase difference versus B_0 enclosed or vacuum wavenumber

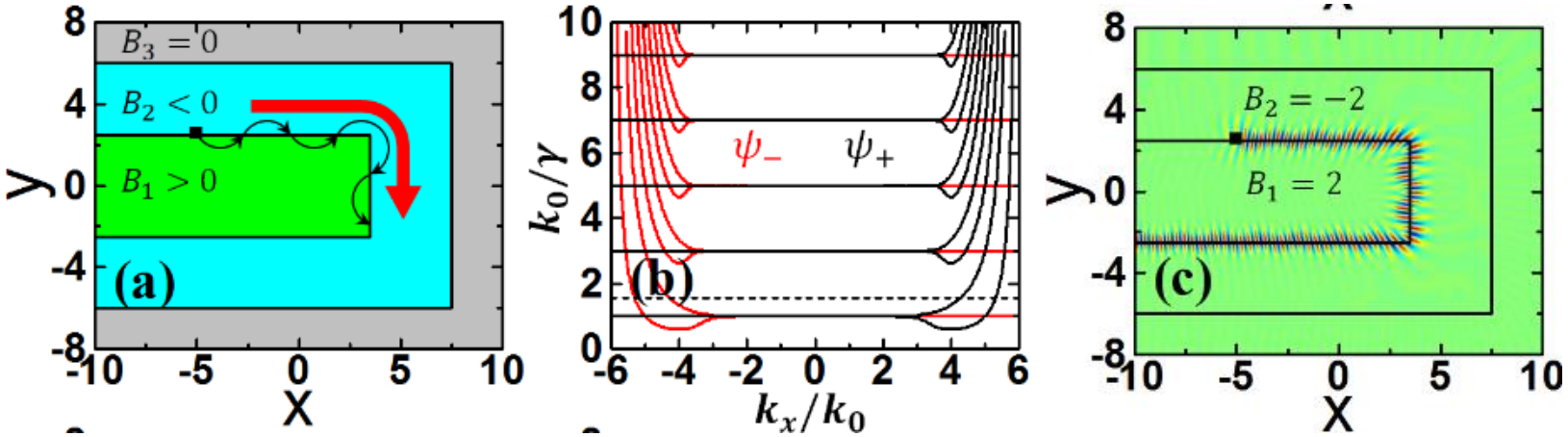
- Interference pattern $|E_z + H_z|$ when $B_0 = 0$



- Interference shifted by varying B_0

One-way edge state

- Wave guiding with magnetic force.
- Interface between regions of pseudo magnetic fields of opposite signs: one-way edge state for photon



- Can travel around sharp corners without backscattering

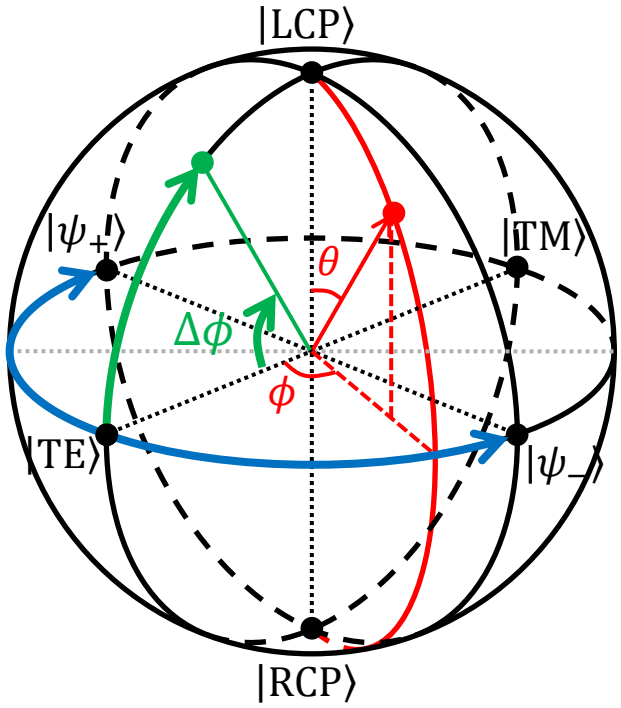
Phys. Rev. Lett. 114, 103902 (2015)

Third choice on Poincaré sphere

Time-reversal operation

CP: $|LCP\rangle \rightarrow |LCP\rangle$
$|RCP\rangle \rightarrow |RCP\rangle$

LP: $|TE\rangle \rightarrow |TE\rangle$
$|TM\rangle \rightarrow |TM\rangle$

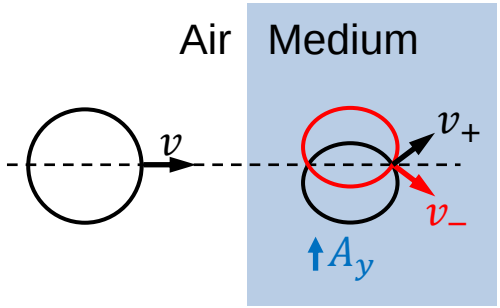


Pseudo-spin devices

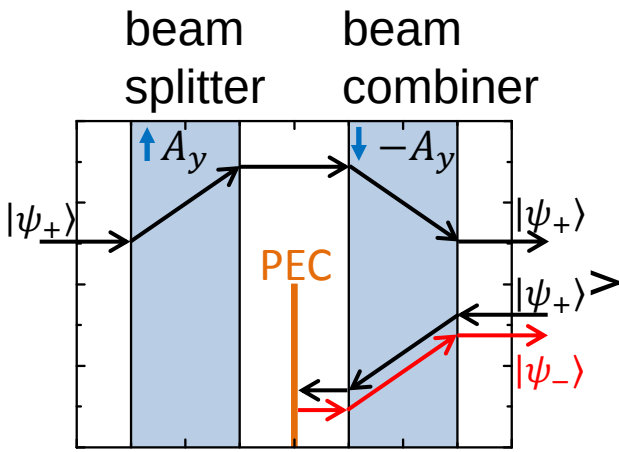
- Green arrow: Pseudo-spin waveplate
- Blue arrow: Pseudo-spin beam splitter

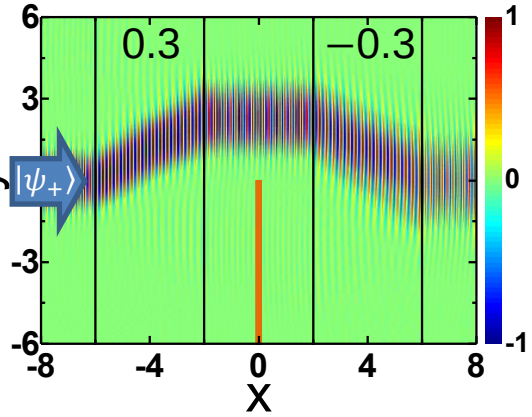
Beam splitter and unidirectional propagation

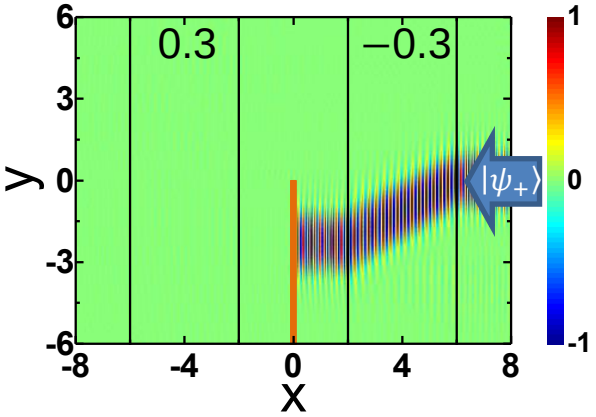
- Constant gauge field A_y splits the local dispersions in y-direction
  - beam splitting of two pseudo-spins



- Unidirectional propagation







Appl. Phys. Lett. 107, 241106 (2015)

Acknowledgement

Metasurface works

University of Birmingham

Shuang Zhang, Guixin Li

Nanjing University

Hui Liu

Baptist University

K W Cheah

City University of Hong Kong

Edwin Pun

PT-symmetry works

Tongji University

Hong Chen

Vienna University of Technology

Stefan Rotter

Group Members

Fu Liu

James Gear

Shiyi Xiao

Pengjiang Wei

Acknowledgement for support
Grant # 630979, EU

