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Pseudospin-1 Physics with Photonic Crystals

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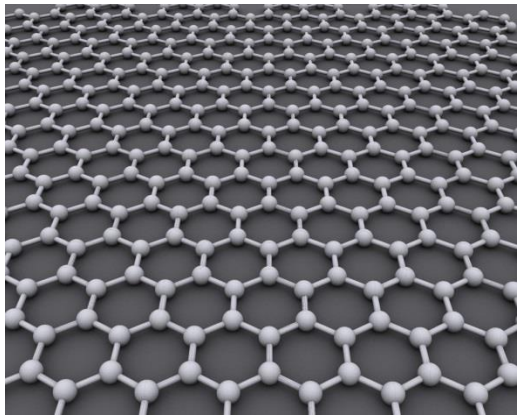
Outline

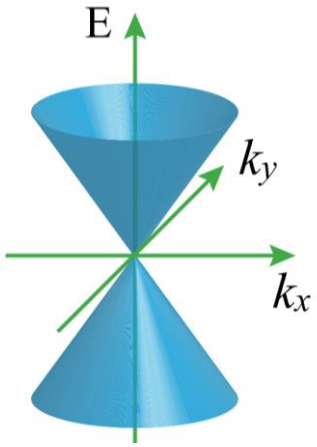
- Background
- Spin-orbit Hamiltonian with pseudospin-1
- Photonic analog of gate voltage in graphene
- Boundary conditions at the interface
- Super Klein tunneling
- Supercollimation in the superlattice of PCs and 1D disorder modulation of PCs
- Localization behavior in 1D disordered Pseudospin-1 system

Background



Graphene [from wiki]

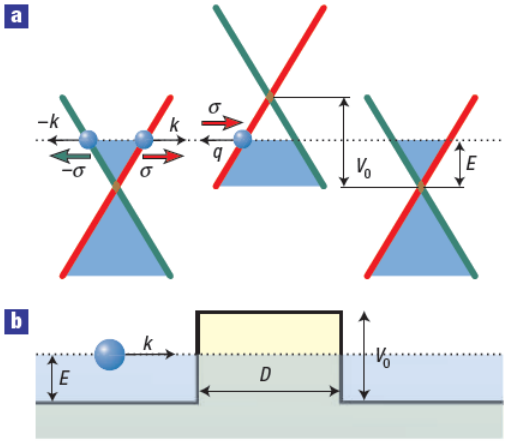
K.S. Novoselov, etc., Science **306**, 666 (2004)

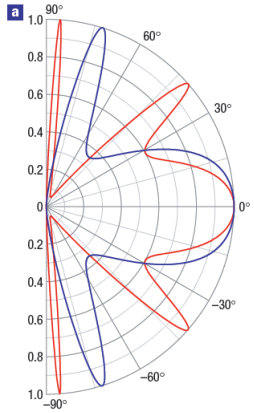


Dirac Cones at the Brillouin Zone Boundary K and K'

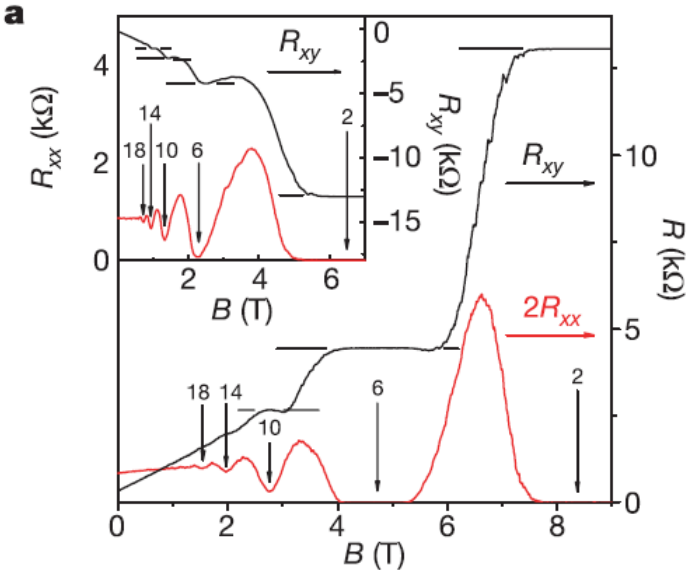
Massless Dirac Equation

$$H = v_F \sigma_x p_x + v_F \sigma_y p_y$$



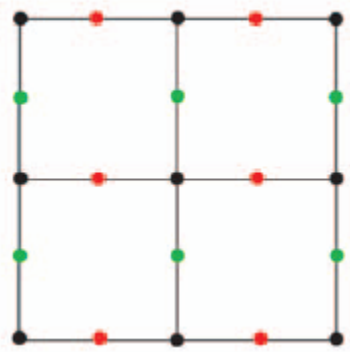


Klein tunneling
M. I. Katsnelson, etc., Nat. Phys. **2**, 620 (2006)

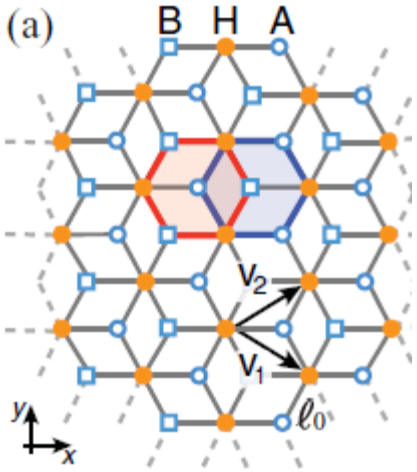


Integer quantum Hall effect
Y. B. Zhang, etc., Nature **438**, 201 (2005)

Pseudospin-1 system with artificial crystals in ultracold atom systems



Line-centered-square optical lattice
R. Shen, etc., Phys. Rev. B **81**, 041410 (2010)

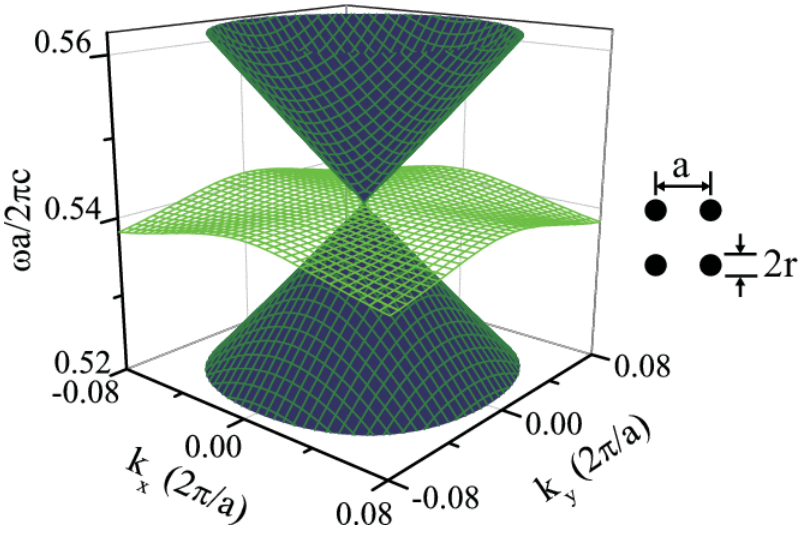


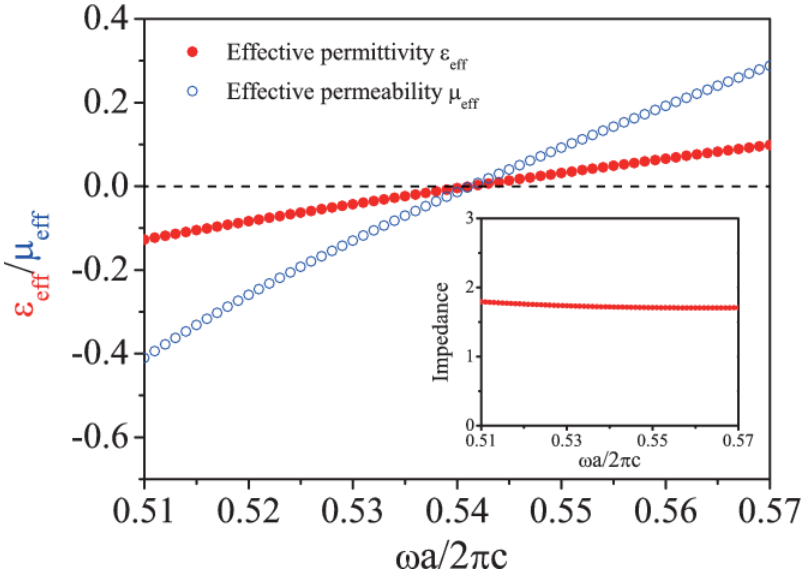
Dice or T3 lattice
D. F. Urban, etc., Phys. Rev. B **84**, 115136(2011)

Problems:

1. Extremely low temperature, around 1 microkelvin
2. Very narrow working band width

Conical dispersion at k=0 for certain 2D photonic crystals





Conical dispersion for a square lattice of dielectric cylinders in air
r = 0.2a, ε = 12.5

ε = μ =0 at the Dirac-like point frequency ω_D

Conical dispersion at k=0 for certain 2D photonic crystals

2D Photonic cystals
Dirac-like cones,
Accidental degeneracy of monopole and dipole excitations



Effective medium crossing $\varepsilon = \mu = 0$ at the Dirac-like point frequency

Problems with Dirac equation (pseudospin $\frac{1}{2}$) description:

1. Omission of the flat band
2. Berry phase is π

Spin-orbit interaction with pseudospin 1 from Maxwell's equations

TE case: Hx, Hy and Ez are nonzero components.

Matrix equation from Maxwell's equations

$$\begin{pmatrix} 0 & -i\frac{\partial}{\partial x}-\frac{\partial}{\partial y} & 0 \\ -i\frac{\partial}{\partial x}+\frac{\partial}{\partial y} & 0 & -i\frac{\partial}{\partial x}-\frac{\partial}{\partial y} \\ 0 & -i\frac{\partial}{\partial x}+\frac{\partial}{\partial y} & 0 \end{pmatrix}\tilde{\psi}=\omega\begin{pmatrix}\mu & 0 & 0\\ 0 & 2\varepsilon & 0\\ 0 & 0 & \mu\end{pmatrix}\tilde{\psi}$$

$$\tilde{\psi}=\begin{pmatrix}-iH_x-H_y\\ E_z\\ iH_x-H_y\end{pmatrix}\quad\text{(Transverse)}$$

$$\tilde{\psi}=\begin{pmatrix}H_x-iH_y\\ E_z\\ -H_x-iH_y\end{pmatrix}\quad\text{(Longitudinal)}\qquad \varepsilon=\mu=0$$

Matrix equation in k space:

$$\begin{pmatrix} 0 & k_x-ik_y & 0 \\ k_x+ik_y & 0 & k_x-ik_y \\ 0 & k_x+ik_y & 0 \end{pmatrix}\tilde{\psi}=(\omega-\omega_D)\begin{pmatrix}\tilde{\mu} & 0 & 0\\ 0 & 2\tilde{\varepsilon} & 0\\ 0 & 0 & \tilde{\mu}\end{pmatrix}\tilde{\psi}$$

Here $\tilde{\varepsilon}=\omega_D\left.\frac{d\varepsilon}{d\omega}\right|_{\omega=\omega_D}$ and $\tilde{\mu}=\omega_D\left.\frac{d\mu}{d\omega}\right|_{\omega=\omega_D}$ from the first order approximation of $\omega\varepsilon$ and $\omega\mu$ near $\omega=\omega_D$:

$$\omega\varepsilon\approx(\omega-\omega_D)\omega_D\left.\frac{d\varepsilon}{d\omega}\right|_{\omega=\omega_D}=(\omega-\omega_D)\tilde{\varepsilon}\qquad \omega\mu\approx(\omega-\omega_D)\omega_D\left.\frac{d\mu}{d\omega}\right|_{\omega=\omega_D}=(\omega-\omega_D)\tilde{\mu}$$

Spin-orbit interaction with pseudo spin 1 from Maxwell's equations

Let $\psi = U\tilde{\psi}$, where $U = \begin{pmatrix} \sqrt{\tilde{\mu}} & 0 & 0 \\ 0 & \sqrt{2\tilde{\varepsilon}} & 0 \\ 0 & 0 & \sqrt{\tilde{\mu}} \end{pmatrix}$, and left multiply U^{-1} on both sides,

$$\frac{1}{\sqrt{2}} v_g \begin{pmatrix} 0 & k_x - ik_y & 0 \\ k_x + ik_y & 0 & k_x - ik_y \\ 0 & k_x + ik_y & 0 \end{pmatrix} \psi = (\omega - \omega_D)\psi \qquad v_g = \frac{c}{n_p + \omega \frac{dn_p}{d\omega}} \approx \frac{1}{\sqrt{\tilde{\varepsilon}\tilde{\mu}}}$$

Let $\delta\omega = \omega - \omega_D$, and S as the spin vector for spin 1.

Spin matrices for spin 1

$$S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad S_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

Spin-orbit Hamiltonian:

$$H\psi = v_g \vec{S} \cdot \vec{k} \psi = \delta\omega\psi$$

Spin-orbit interaction with pseudospin 1 from Maxwell's equations

The eigen vectors for the Hamiltonian,

$$\bar{\psi}_s^{(k)} = \frac{1}{2}\begin{pmatrix} se^{-i\theta_k} \\ \sqrt{2} \\ se^{i\theta_k} \end{pmatrix} \quad (\delta\omega = sv_g k, s = \pm 1) \qquad \bar{\psi}_s^{(k)} = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta_k} \\ 0 \\ -e^{i\theta_k} \end{pmatrix} \quad (\delta\omega = sv_g k, s = 0)$$

In the two eigenvectors with $s = \pm 1$, electric field $E_z^{(k)}$ is set to be unity.

For arbitrary $E_z^{(k)}$, we have $\psi_T^{(k)} = E_z^{(k)} \bar{\psi}_{\pm 1}^{(k)}$.

Eigen vectors in terms of electromagnetic fields:

$$\psi_T^{(k)} = \frac{1}{2}\begin{pmatrix} -\sqrt{\tilde{\mu}/\tilde{\varepsilon}}(iH_x^{(k)} + H_y^{(k)}) \\ \sqrt{2}E_z^{(k)} \\ \sqrt{\tilde{\mu}/\tilde{\varepsilon}}(iH_x^{(k)} - H_y^{(k)}) \end{pmatrix} \quad (\mathrm{s} = \pm 1) \qquad \psi_L^{(k)} = \begin{pmatrix} H_x^{(k)} - iH_y^{(k)} \\ 0 \\ -H_x^{(k)} - iH_y^{(k)} \end{pmatrix} \quad (s = 0)$$

Berry phase:

$$\gamma = i\oint \langle \psi_{\mathbf{k}} | \nabla_{\mathbf{k}} | \psi_{\mathbf{k}} \rangle \cdot d\mathbf{k} = 0$$

Photonic analog of gate voltage

Length scale of photonic crystal changes:

$$a \to \bar{a} = \alpha a$$
$$r \to \bar{r} = \alpha r$$

Frequency and wave vector change

$$\omega \to \bar{\omega} = \frac{1}{\alpha}\omega$$
$$k \to \bar{k} = \frac{1}{\alpha}k$$

$$\bar{v}_g = \nabla_{\bar{k}}\bar{\omega} = \nabla_{\frac{1}{\alpha}k}(\frac{1}{\alpha}\omega) = \nabla_k \omega = v_g$$

Hamitonian with 1D variation of potential V(x)

$$H = v_g \vec{S}\cdot\vec{k} + V(x)\,\mathrm{I}$$

I is the 3X3 identity matrix

No backscattering for normal incidence → Impedance match near the Dirac-like point frequency ω_D

Boundary conditions from spin-1 Hamiltonian

Assuming the wave function $\psi = (\psi_1 \quad \psi_2 \quad \psi_2)^T$, we can have three differential equations from the pseudospin-1 Hamiltonian,

$$\frac{v_g}{\sqrt{2}}\left(-i\frac{\partial \psi_2}{\partial x} - \frac{\partial \psi_2}{\partial y}\right) = \left[\delta\omega - V(x)\right]\psi_1$$

$$\frac{v_g}{\sqrt{2}}\left(-i\frac{\partial \psi_1}{\partial x} + \frac{\partial \psi_1}{\partial y} - i\frac{\partial \psi_3}{\partial x} - \frac{\partial \psi_3}{\partial y}\right) = \left[\delta\omega - V(x)\right]\psi_2$$

$$\frac{v_g}{\sqrt{2}}\left(-i\frac{\partial \psi_2}{\partial x} + \frac{\partial \psi_2}{\partial y}\right) = \left[\delta\omega - V(x)\right]\psi_3$$

Integrate the first equation from $x_0 - \epsilon$ to $x_0 + \epsilon$ ($x = x_0$ is the interface), and take the limit as $\epsilon \to 0$,

$$-i\frac{v_g}{\sqrt{2}}\int_{x_0-\epsilon}^{x_0+\epsilon}\frac{\partial \psi_2}{\partial x}dx - \frac{v_g}{\sqrt{2}}\int_{x_0-\epsilon}^{x_0+\epsilon}\frac{\partial \psi_2}{\partial y}dx = \int_{x_0-\epsilon}^{x_0+\epsilon}\left[\delta\omega - V(x)\right]\psi_1 dx$$

Assume $V(x)$ and the wave functions are finite,

$$\psi_2(x_0+\epsilon) = \psi_2(x_0-\epsilon), \text{ i.e., } \psi_2 \text{ is continuous at the boundary.}$$

Boundary conditions from spin-1 Hamiltonian

Similarly,

$$\psi_1(x_0+\epsilon)+\psi_3(x_0+\epsilon)=\psi_1(x_0-\epsilon)+\psi_3(x_0-\epsilon),$$

i.e., $\psi_1+\psi_3$ is continuous at the boundary, and

$$\left[\delta\omega-V(x)\right](\psi_3-\psi_1) \text{ is continuous.}$$

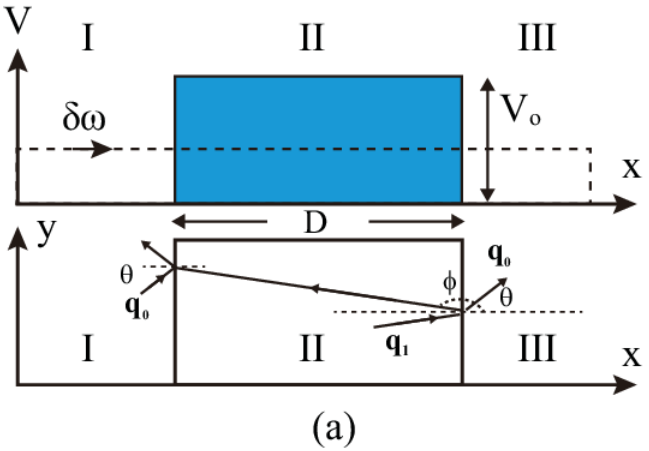
Apply the three boundary conditions from spin=1 Hamiltonian to eigen functions in terms of EM fields in TE case, we have the following correspondence,

$$\psi_2 \text{ is continuous } \Rightarrow E_z \text{ is continuous.}$$

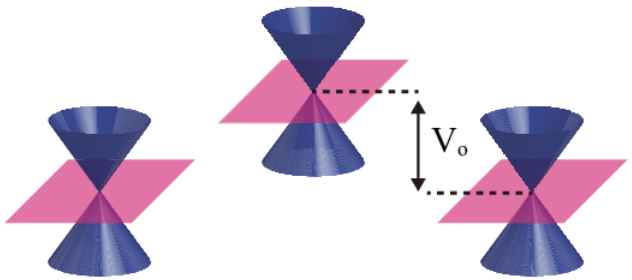
$$\psi_1+\psi_3 \text{ are continuous } \Rightarrow \mathrm{H}_y \text{ is continuous.}$$

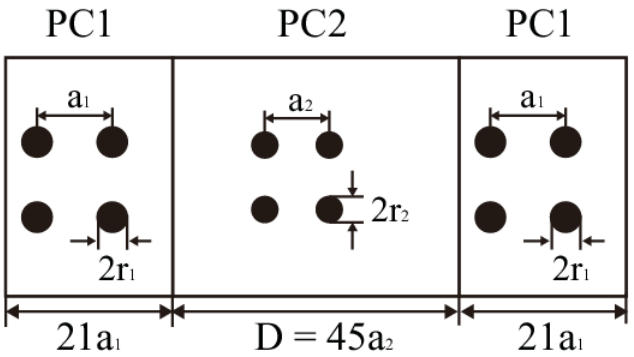
$$\left[\delta\omega-V(x)\right](\psi_3-\psi_1) \text{ is continuous } \Rightarrow \mathrm{B}_x \text{ is continuous.}$$

Klein tunneling for a square potential barrier



(a)





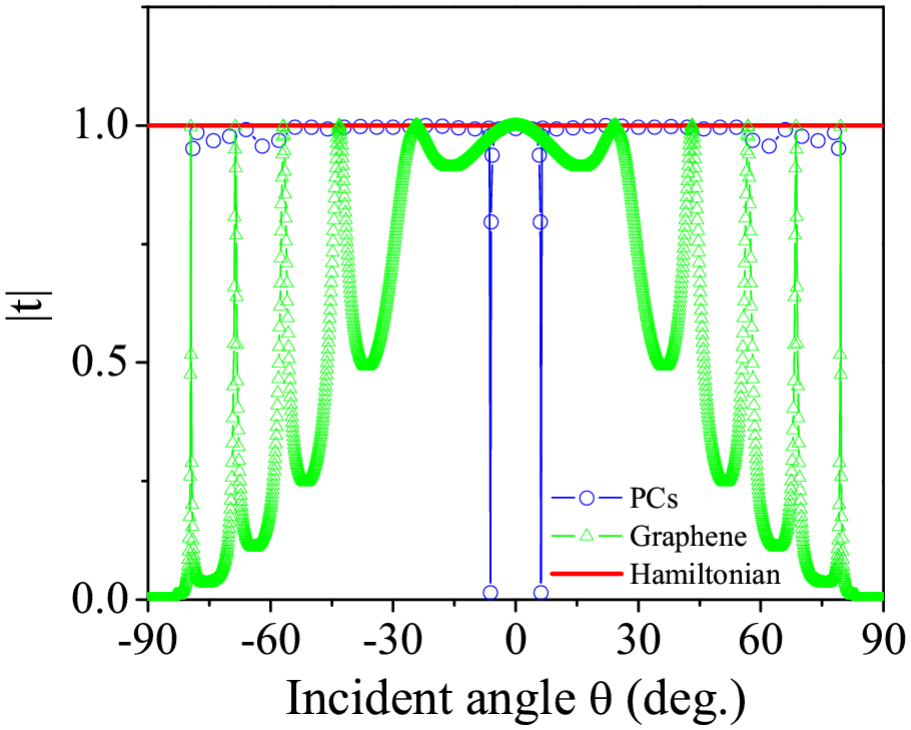
(b)

$$T_{ph} = \frac{\cos^2\theta_k \cos^2\theta_q}{\cos^2 q_x D \cos^2\theta_k \cos^2\theta_q + \frac{\sin^2 q_x D}{4}(\cos^2\theta_q + \cos^2\theta_k)^2}$$

θ_k and θ_q are angles of wave vector k and q in region I (III) and II, respectively. $\delta\omega = \omega - \omega_D$. T=1 for all incident angles θ when $\delta\omega = V_0/2$.

$$a' = 15a/14 \qquad r' = 15r/14$$

Klein tunneling for a square potential barrier



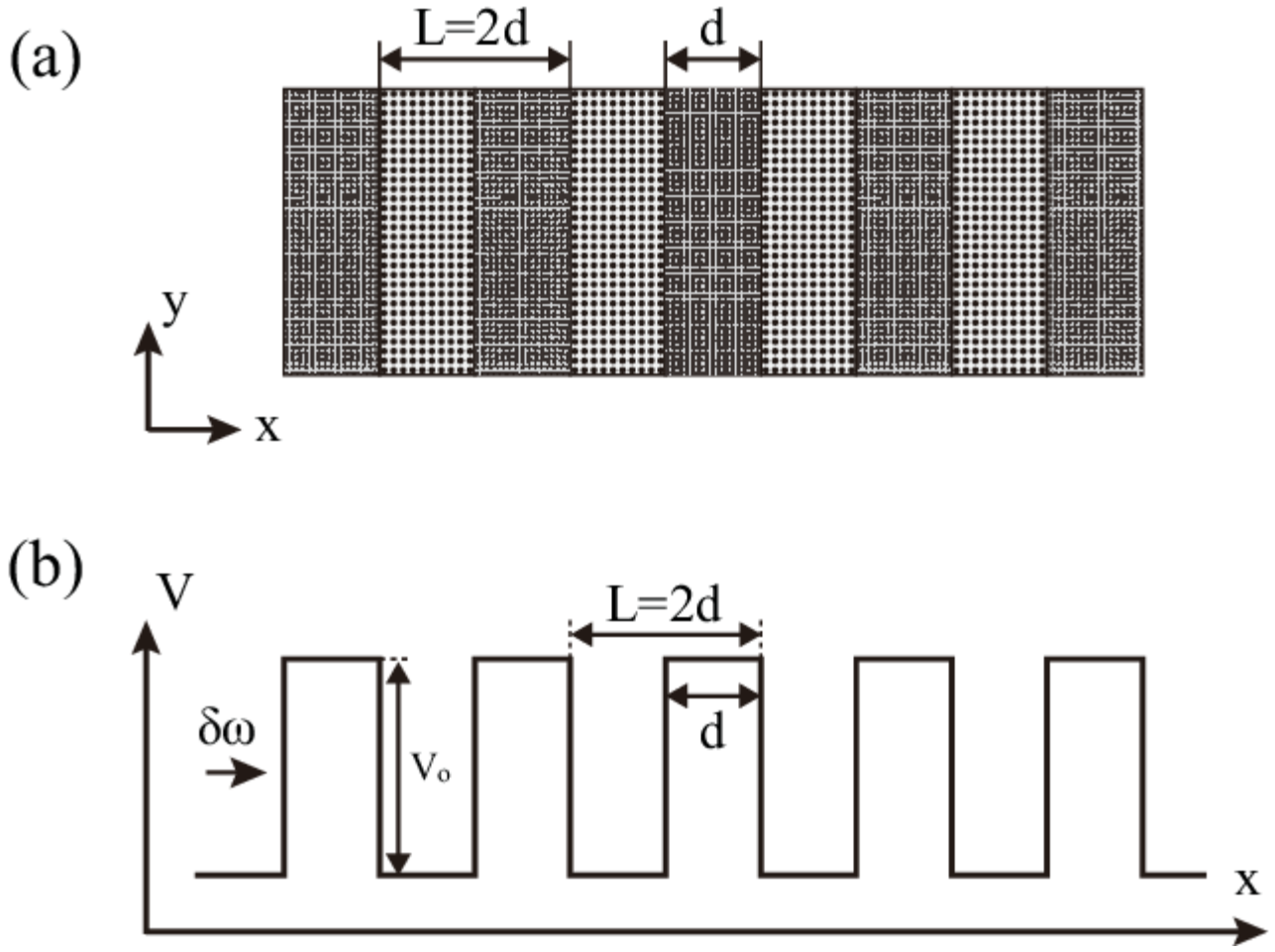
The photon potential , frequency and group velocity:

$$V_0 = \omega_{D2}/15 \qquad \delta\omega = V_0/2 \qquad v_g = 0.2962c$$

The electron potential and energy:

$$E/\hbar v_F = \delta\omega / v_g \qquad V/\hbar v_F = V_0 / v_g$$

1D superlattice of photonic crystals



Kronig-Penney type photonic potential

Band structure of 1D superlattice

Dispersion relation from TMM method:

$$2\cos k_x L - 2\cos q_{2x} d \cos q_{1x}(L-d) + ss' \sin q_{2x} d \sin q_{1x}(L-d)\,\varphi\left(\theta_{q_1}, \theta_{q_2}\right) = 0$$

$$q_{1x}^2 + k_y^2 = \left((\delta\omega + \frac{V_0}{2})/v_g\right)^2$$

$$q_{2x}^2 + k_y^2 = \left[(\delta\omega - V_0/2)/v_g\right]^2$$

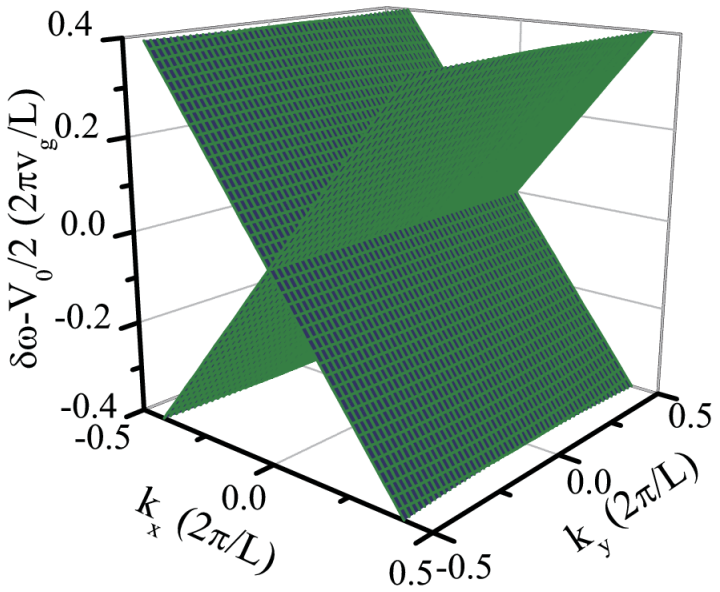
$$\varphi\left(\theta_{q_1}, \theta_{q_2}\right) = \frac{\cos\theta_{q_1}}{\cos\theta_{q_2}} + \frac{\cos\theta_{q_2}}{\cos\theta_{q_1}}$$

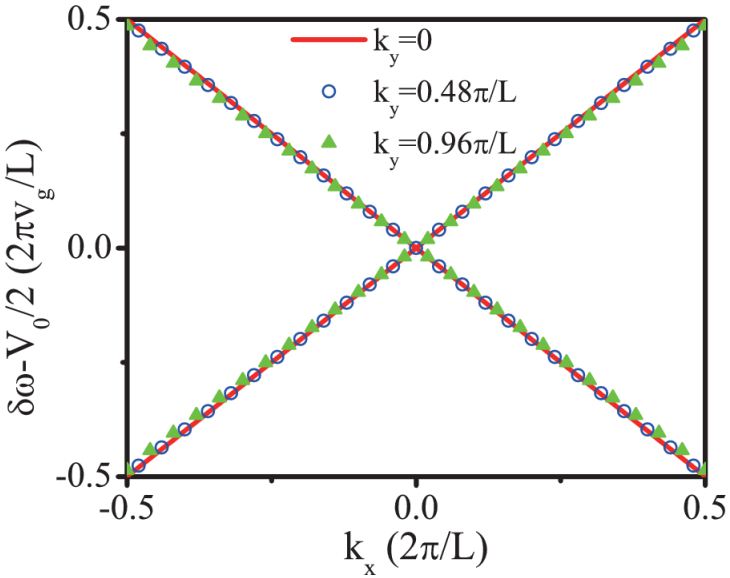
k_x is the Bloch wave vector and k_y is the y component of the wave vector.

When $L = 2d$ and $\delta\omega = 0$, we have $k_x = 0$ independent of k_y and V_0 ($V_0 \neq 0$).

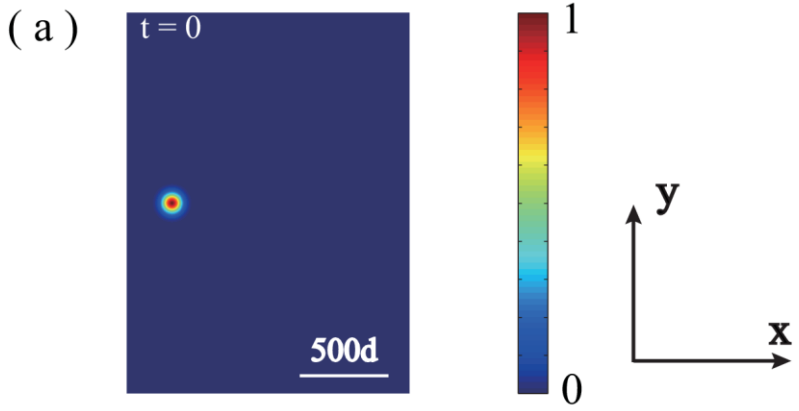
Near $\delta\omega = 0$ and for small k_y, the band dispersion can be expanded as

$$\delta\omega = s v_g |k_x| \qquad s = \mathrm{sgn}(\delta\omega)$$





Super collimation in 1D superlattice of photonic crystals



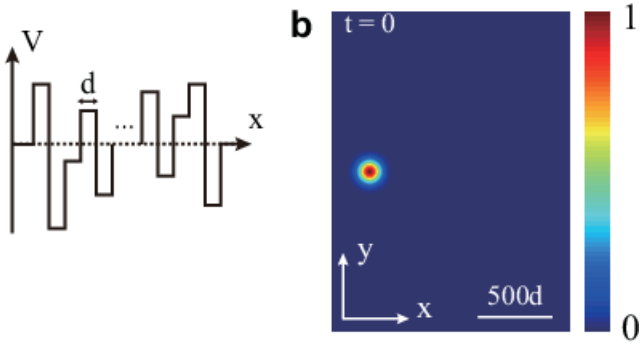
Superlattice realized by stacking layers of PCs, PC1 and PC2 with equal thickness.

$$d = 15a,\ L = 2d = 30a$$

$$\delta\omega = 0.06\pi v_g / L$$

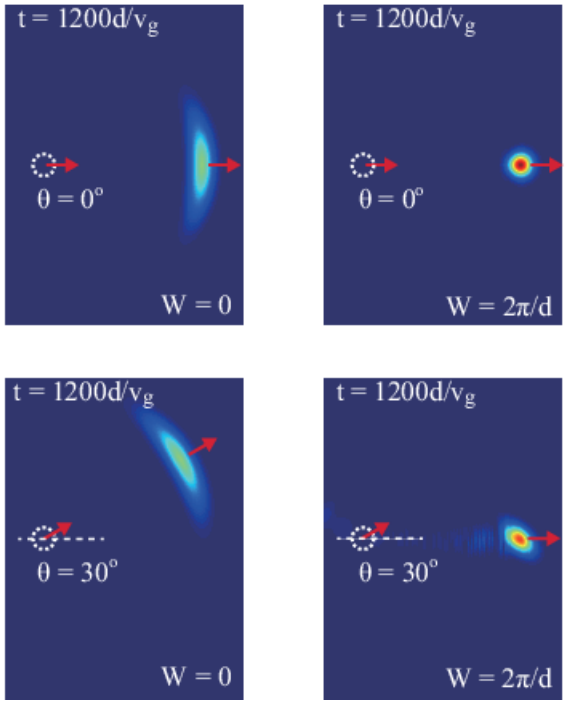
a is the lattice constant of PC2.

Super collimation in 1D randomized media



Reduced enter frequency:
$\delta\omega_c = 0.015\pi v_g/d$

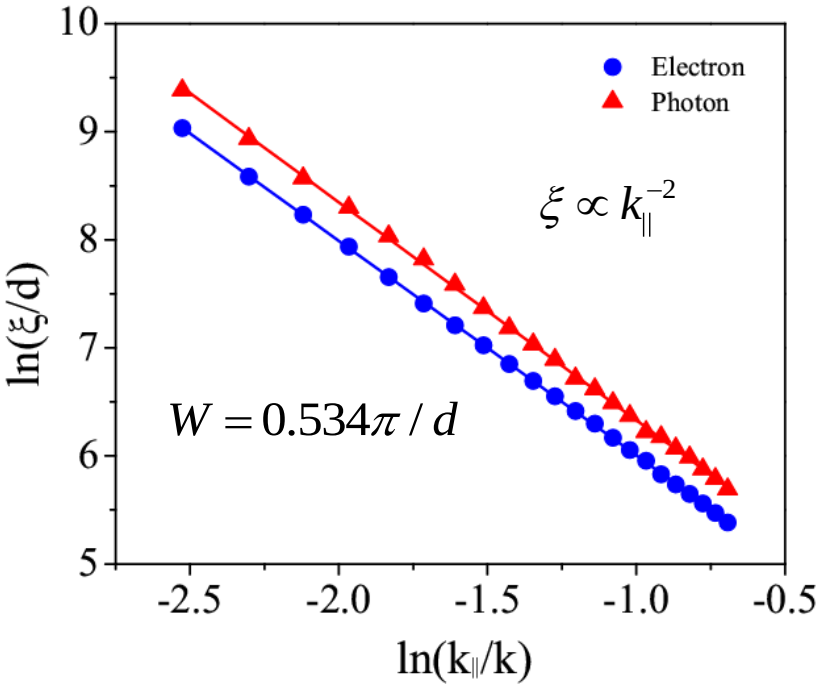
Half width of the initial Gaussian wave packet:
$r_0 = 40d$



1D disordered
photonic potential

Uniform distribution:
$\bar{V} \in [-W, W]$ $\bar{V} \equiv V/v_g$
W: randomness strength

Localization length as a function of $k_{||}$



Localization length:

$$\xi = \frac{2(L_2 - L_1)}{< \ln T_1 > - < \ln T_2 >}$$

T_1 and T_2 are transmission coefficients for the sample thicknesses L_1 and L_2, respectively. <> indicates the ensemble average for the disordered system.

$\bar{V} \equiv V_0 / v_g$ for photons and $\bar{V} \equiv \mathrm{U}_0 / \hbar v_F$ for electrons, where V_0 and U_0 are the photonic and electronic potentials, respectively. $\bar{V} \in [-W, W]$.

d: the thickness of one layer

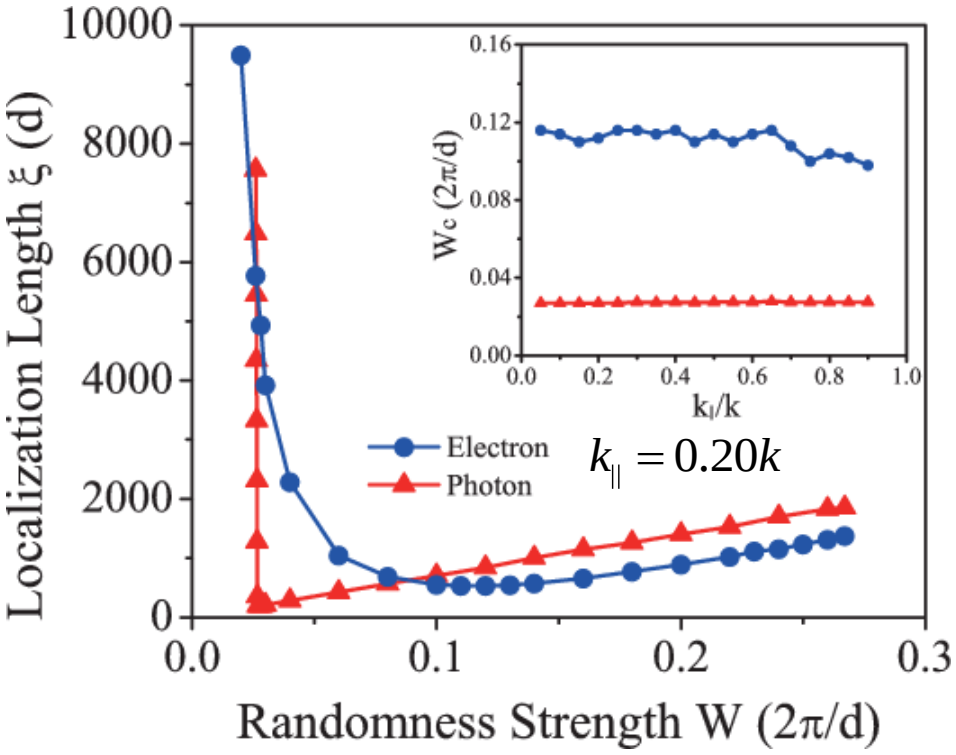
$k_{||}$: the wave vector component parallel to the interface

$\delta\omega = \omega - \omega_D$: the reduced frequency for psedospin-1 photons

E: the electron energy in graphene

$\delta\omega / v_g = E / \hbar v_F = 0.0534\pi / d$

Localization length as a function of randomness strength

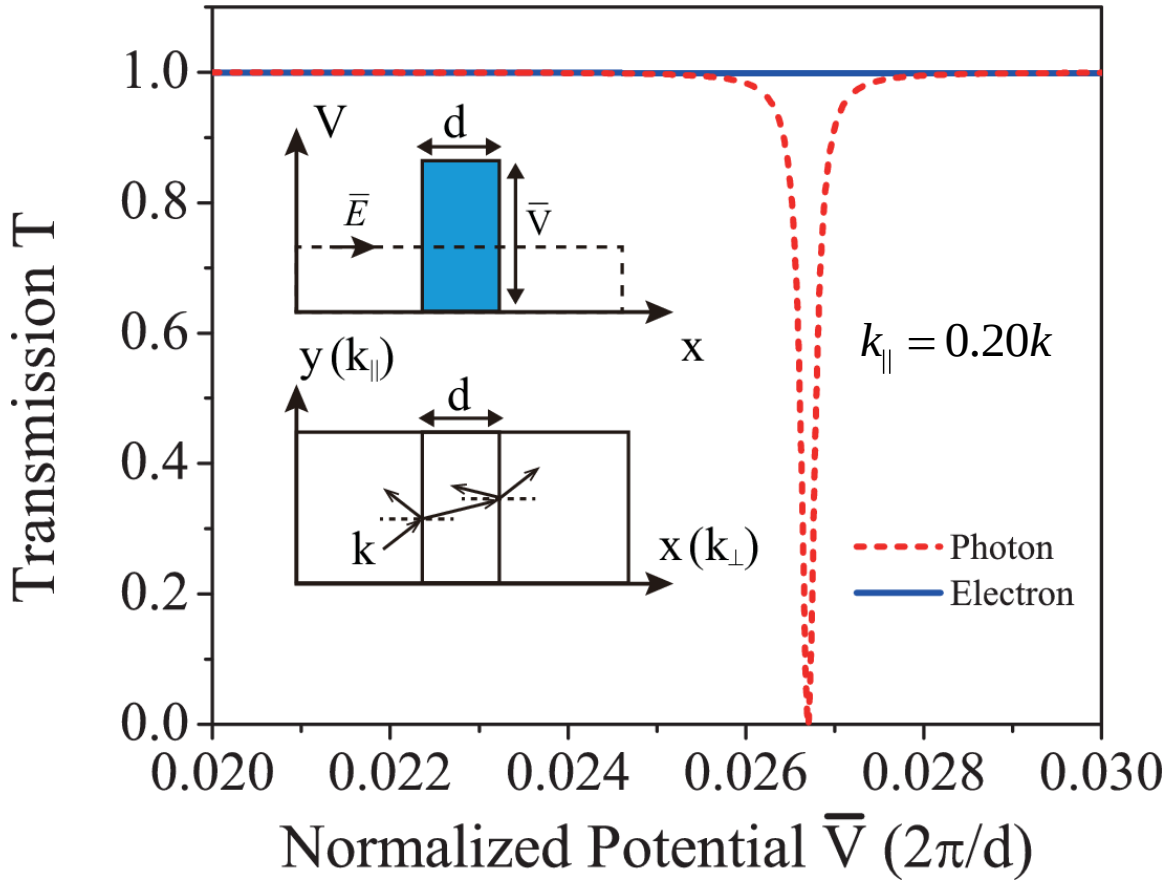


$$\delta\omega / v_g = E / \hbar v_F = 0.0534\pi / d$$

$$V_0 / v_g \text{ and } V_e / \hbar v_F \in [-W,\ W]$$

Minimum localization length exists for a critical value of randomness strength W_c.

Transmission as a function of normalized potential

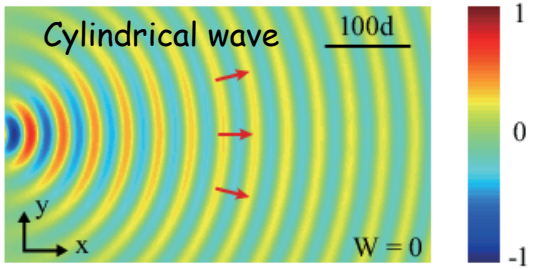


$$V_0 / v_g = V_e / \hbar v_F = \bar{V} \qquad \delta\omega / v_g = E / \hbar v_F = \bar{E}$$

$$\psi_{ph}^{(k)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ for photons } k_y \neq 0 \text{ at the Dirac-like point}$$

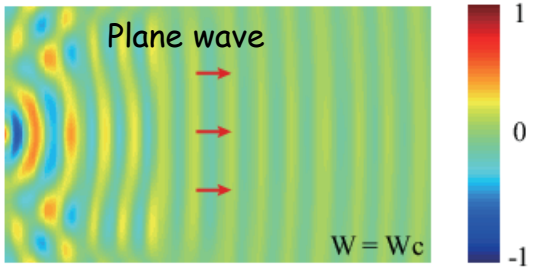
$$\psi_{el}^{(k)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ for electrons } k_y \neq 0 \text{ at the Dirac point}$$

Wave filter



Single PC

Half width of beam: $4d$



1D disordered pseudospin-1 system

$$\overline{\delta\omega} = 0.0534\pi / d$$

Summary

1. We demonstrated that certain dielectric photonic crystals exhibiting Dirac-like conical dispersion at k=0 can be used to realize photonic pseudospin-1 materials.
2. Photonic potential can be varied by a simple change of length scale of PCs.
3. Super Klein tunneling effect for pseudospin-1 photons.
4. Super collimation behaviors for 1D PC superlattice and disordered system.
5. Unusual localization behavior for pseudospin-1 photons in 1D disordered system.