Topological Description for Photonic Mirrors

Hong Chen

School of Physics, Tongji University, Shanghai , China

同舟共济

Collaborators:

- Dr. Wei Tan, Dr. Yong Sun, Tongji Uni.
- Prof. Shun-Qing Shen , The University of Hong Kong

HKUST IAS, Jan. 11, 2016

outline

1. Introduction
   - topological Insulators, band inversion
   - photonic analogs, mirrors as photonic insulators
2. Topological description: Theoretical study
   - mapping 1D Maxwell's equations to Dirac equation
   - topological orders for photonic mirrors
3. Topological description: Experimental study
   - band inversion, microwave experiments
   - edge modes, microwave and visible light experiments
4. Summary

1. Introduction

M. Z. Hasan *et al.*, *Rev. Mod. Phys.* **82**, 3045 (2010)

topological description for electronic insulators

electronic medium

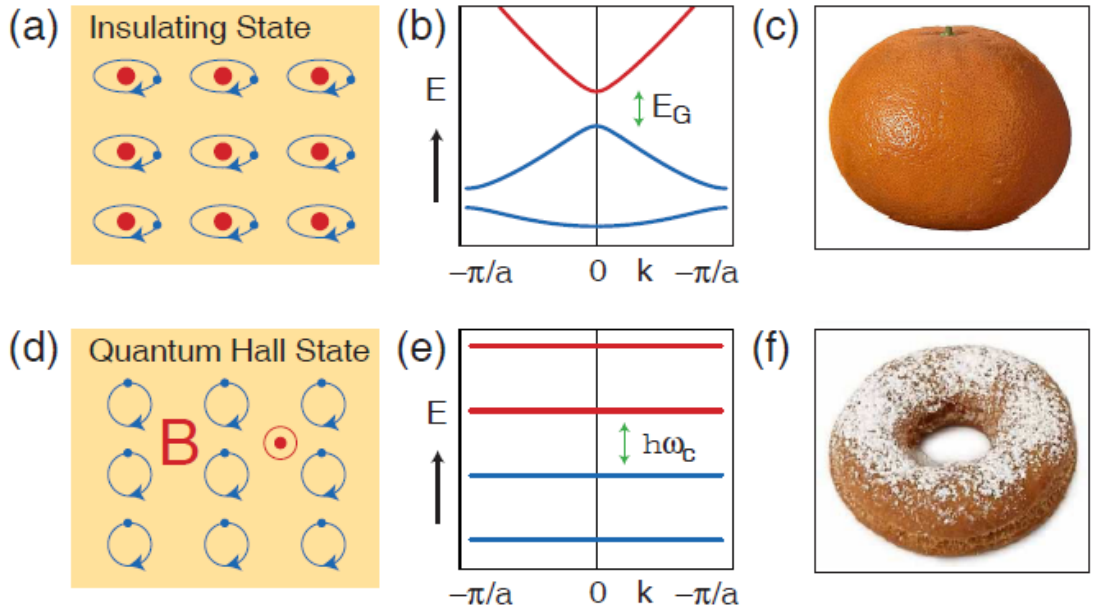
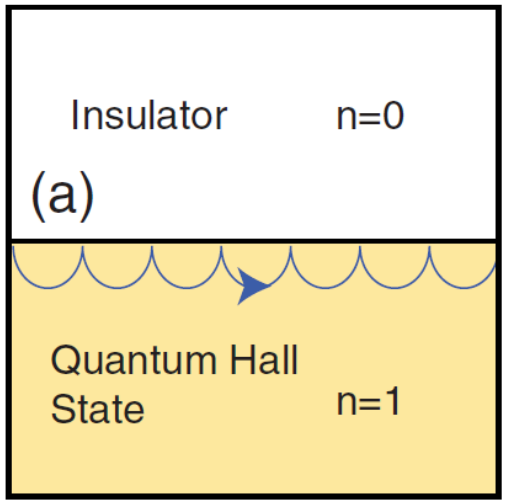


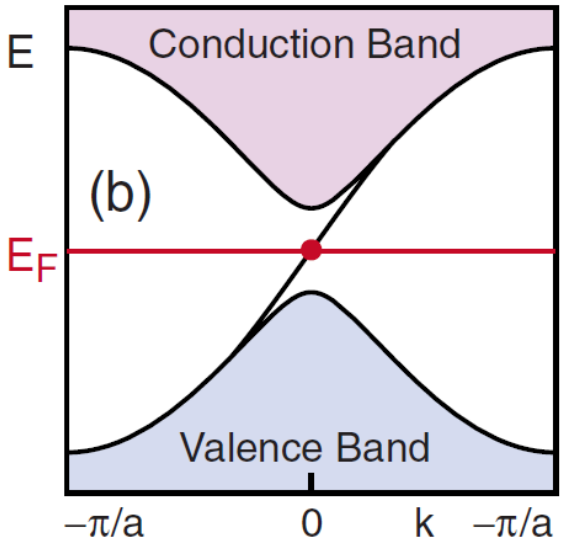
FIG. 1 (a, b, c) The insulating state. (a) depicts an atomic insulator, while (b) shows a simple model insulating band structure. (d, e, f) The quantum Hall state. (d) depicts the cyclotron motion of electrons, and (e) shows the Landau levels, which may be viewed as a band structure. (c) and (f) show two surfaces which differ in their genus, g. $g = 0$ for the sphere (c) and $g = 1$ for the donut (f). The Chern number n that distinguishes the two states is a topological invariant similar to the genus.

Metallic, unidirectional **Edge state**

between insulators with different topological orders

M. Z. Hasan *et al.*, *Rev. Mod. Phys.* (2010)

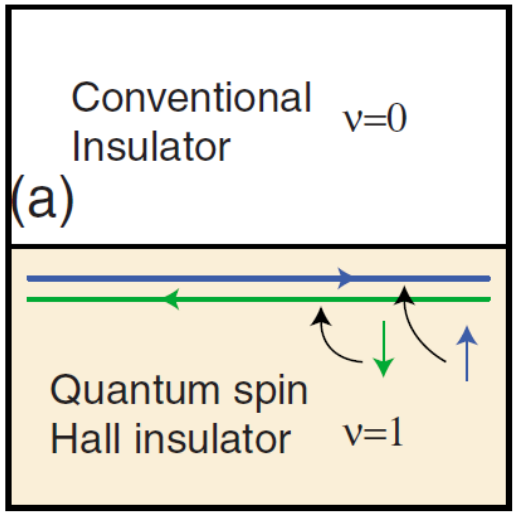


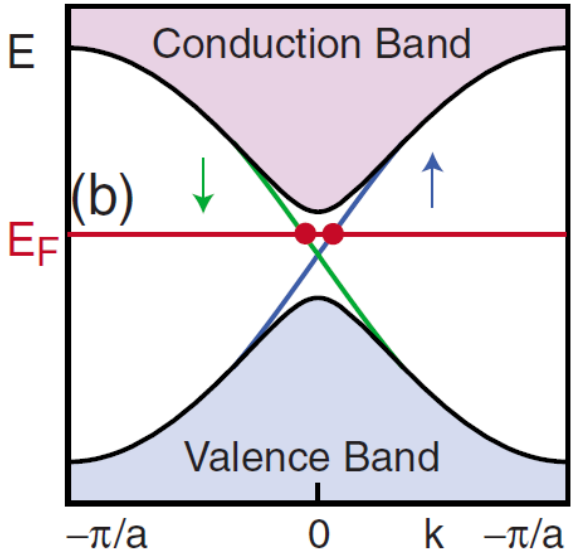


Quantum Hall Effect

Chiral Edge State with *H*

Broken T-symmetry





Quantum Spin Hall Insulator

Edge states without *H*

with T-symmetry

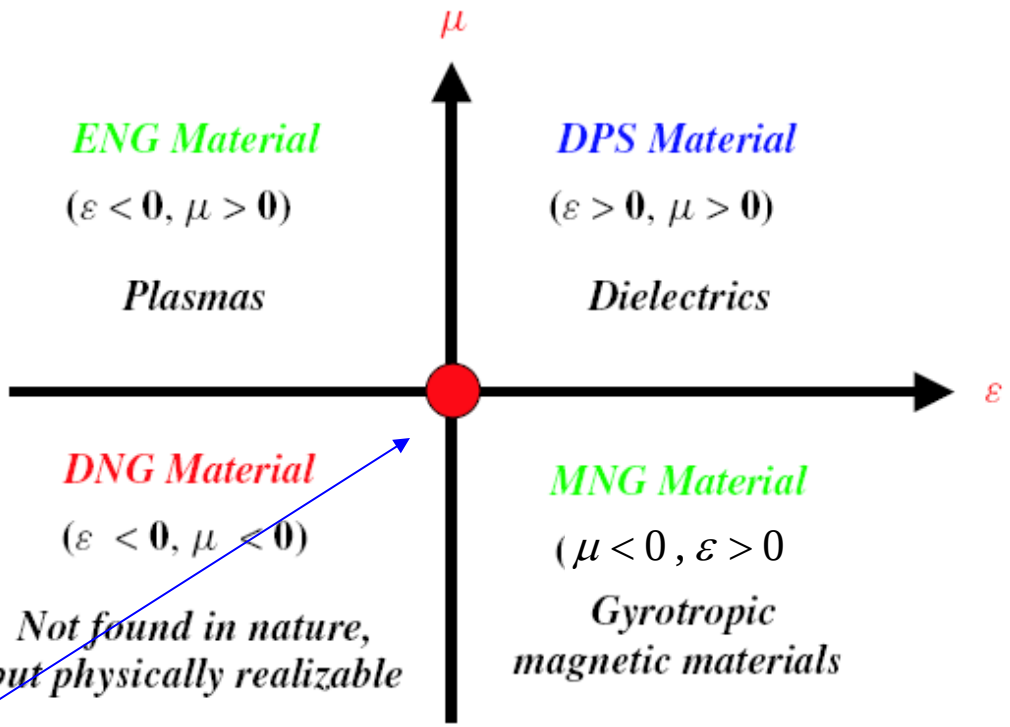
photonic medium: metamaterials with designed ε and μ

Materials Responds To EM Waves:
Permittivity ε and Permeabilty μ

$$n = \pm\sqrt{\varepsilon\mu}$$

single-negative ε-negative (ENG) evanescent wave "barrier"

double-positive (DPS) forward-wave propagation



double-negative (DNG) backward-wave propagation

single-negative μ-negative (MNG) evanescent wave "barrier"

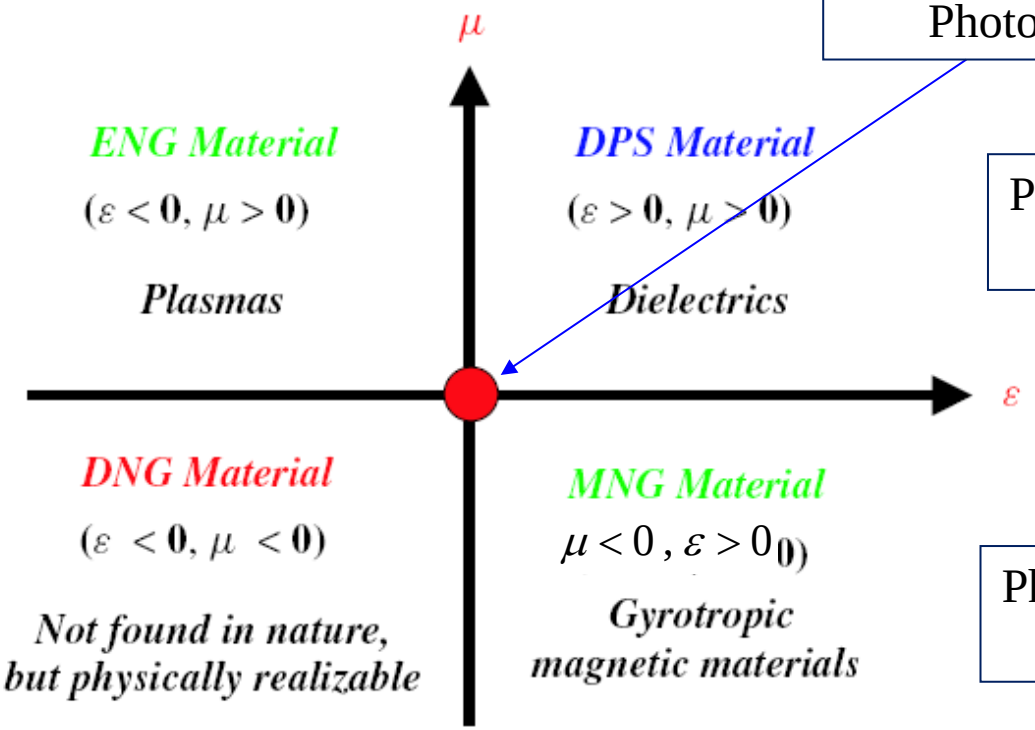
zero-index materials

From ENGHETA and.ZIOLKOWSKI 2006

analog to electrons

Materials Responds To EM Waves:
Permittivity ε and Permeabilty μ

$$n = \pm\sqrt{\varepsilon\mu}$$



Dirac-Point Related Medium
Photonic Graphene?

Photonic Insulator I
ENG Mirror

Photonic Conductor
Right-Handed

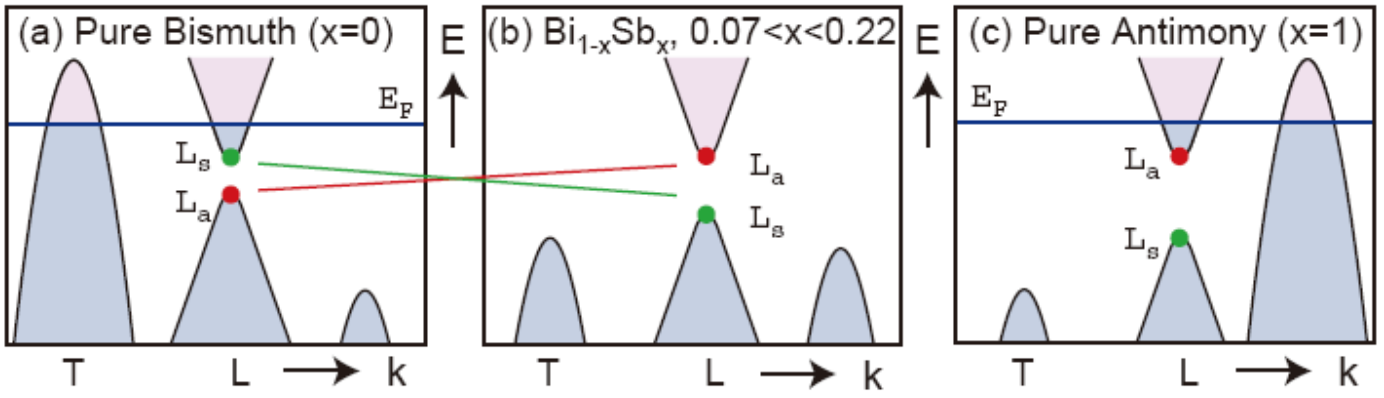
Photonic Conductor
Left-Handed

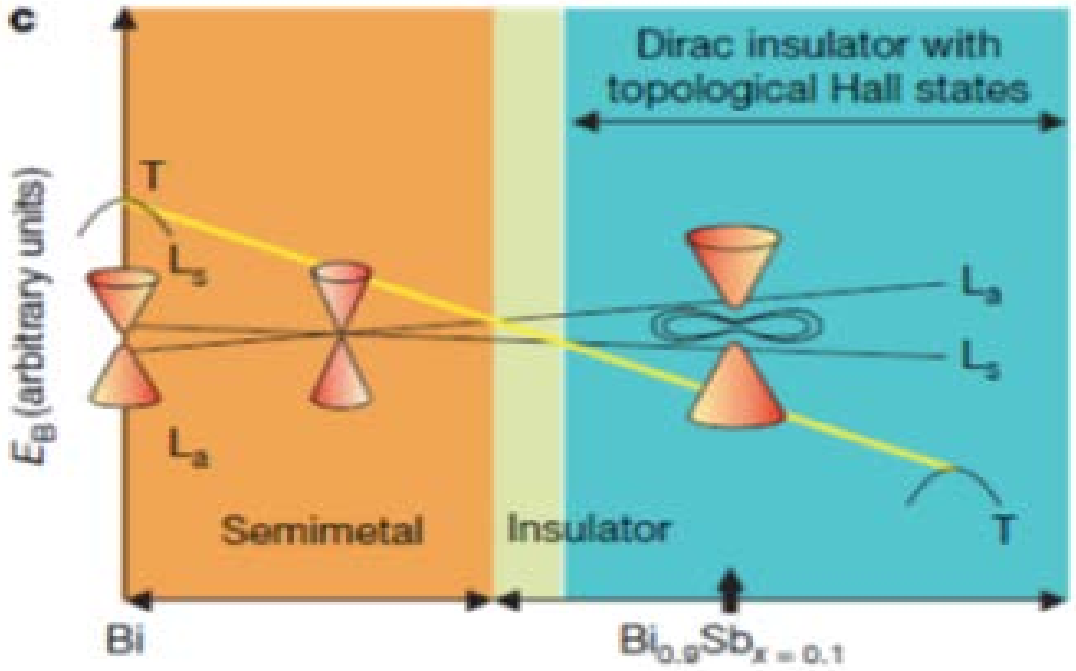
Photonic Insulator II
MNG Mirror

Q1：different topological orders between EMG and MNG mirrors ??

Manipulating topological order: Electronic band inversion transition from normal insulator (NI) to TI [d=3]

M. Z. Hasan *et al.*, *Rev. Mod. Phys.* (2010)





semimetal for
$x < .07$
semiconductor for
$.07 < x < .22$
semimetal for
$x > .18$
bands $L_{s;a}$ invert at
$x \sim .04$

Photonic analog

PRL **100,** 013904 (2008) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2008

Possible Realization of Directional Optical Waveguides in Photonic Crystals with Broken Time-Reversal Symmetry

F. D. M. Haldane and S. Raghu*

Department of Physics, Princeton University, Princeton, New Jersey 08544-0708, USA

(Received 23 March 2005; revised manuscript received 30 May 2007; published 10 January 2008)

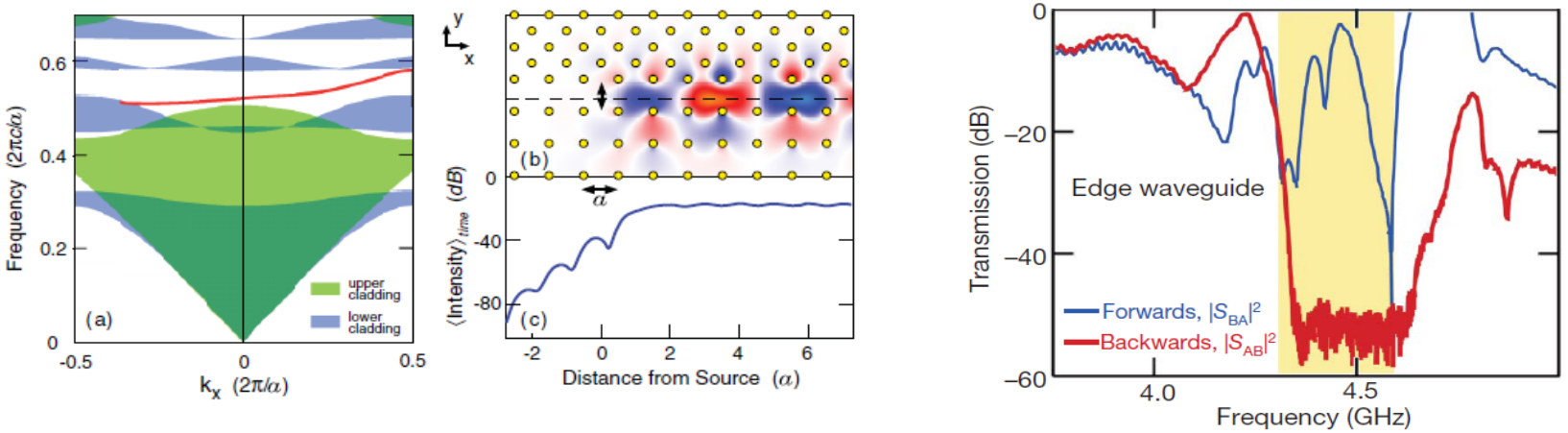
PRL **100,** 013905 (2008) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2008

Reflection-Free One-Way Edge Modes in a Gyromagnetic Photonic Crystal

Zheng Wang, Y. D. Chong, John D. Joannopoulos, and Marin Soljačić

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

(Received 19 June 2007; revised manuscript received 29 October 2007; published 10 January 2008)



Experiments: Wang *et al.*, *Nature* **461**, 772 (2009).

Photonic analog

Photonic topological insulators

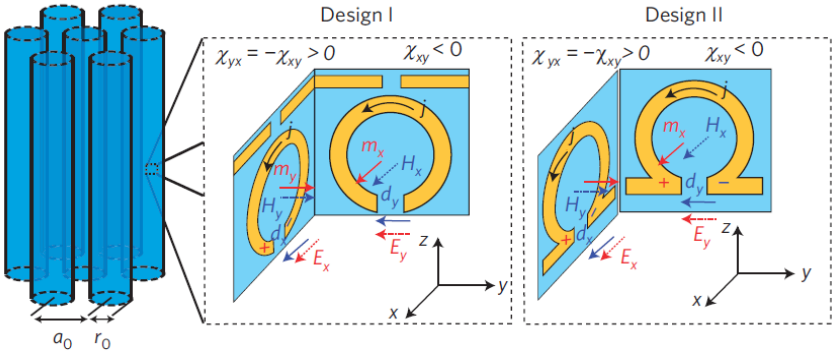
Nature materials 2012

Alexander B. Khanikaev*†, S. Hossein Mousavi†, Wang-Kong Tse, Mehdi Kargarian, Allan H. MacDonald and Gennady Shvets*

Chirality

$$\mathbf{D} = \hat{\epsilon} \cdot \mathbf{E} + i\hat{\chi} \cdot \mathbf{H}$$

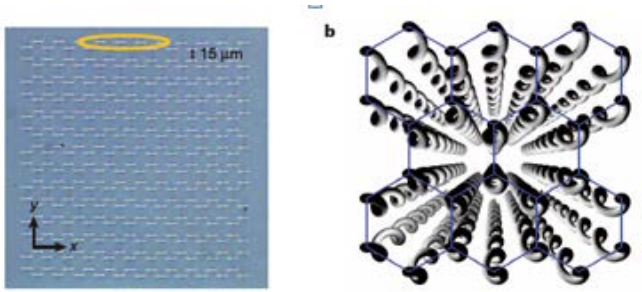
$$\mathbf{B} = \hat{\mu} \cdot \mathbf{H} - i\hat{\chi}^{\mathrm{T}} \cdot \mathbf{E}$$



Photonic Floquet topological insulators

Mikael C. Rechtsman[1]*, Julia M. Zeuner[2]*, Yonatan Plotnik[1]*, Yaakov Lumer[1], Daniel Podolsky[1], Felix Dreisow[2], Stefan Nolte[2], Mordechai Segev[1] & Alexander Szameit[2]

196 | NATURE | VOL 496 | 11 APRIL 2013



Topological photonics

Ling Lu*, John D. Joannopoulos and Marin Soljačić

nature photonics
REVIEW ARTICLE
PUBLISHED ONLINE: 26 OCTOBER 2014 | DOI: 10.1038/NPHOTON.2014.248

Mapping between electrons and photons

Schroedinger Equation : $\hat{H}\psi = E\psi$ $\hat{H} = -\frac{\hbar^2}{2m_e}\nabla^2 + V(\vec{r})$

Maxwell's Equation : $\hat{\Theta}\vec{H} = \left(\omega/c\right)^2 \vec{H}$ $\hat{\Theta} = \nabla \times \left(\frac{1}{\varepsilon(\vec{r})}\nabla \times\right)$

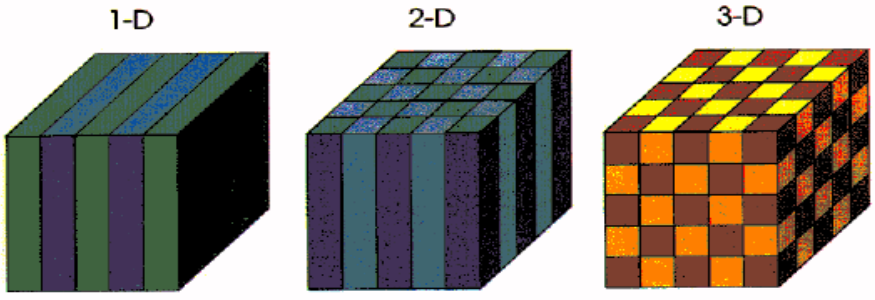
Periodical Structures:

$V(\vec{r} + \vec{R}) = V(\vec{r})$

electronic band gaps , electron insulators

$\varepsilon(\vec{r} + \vec{R}) = \varepsilon(\vec{r})$

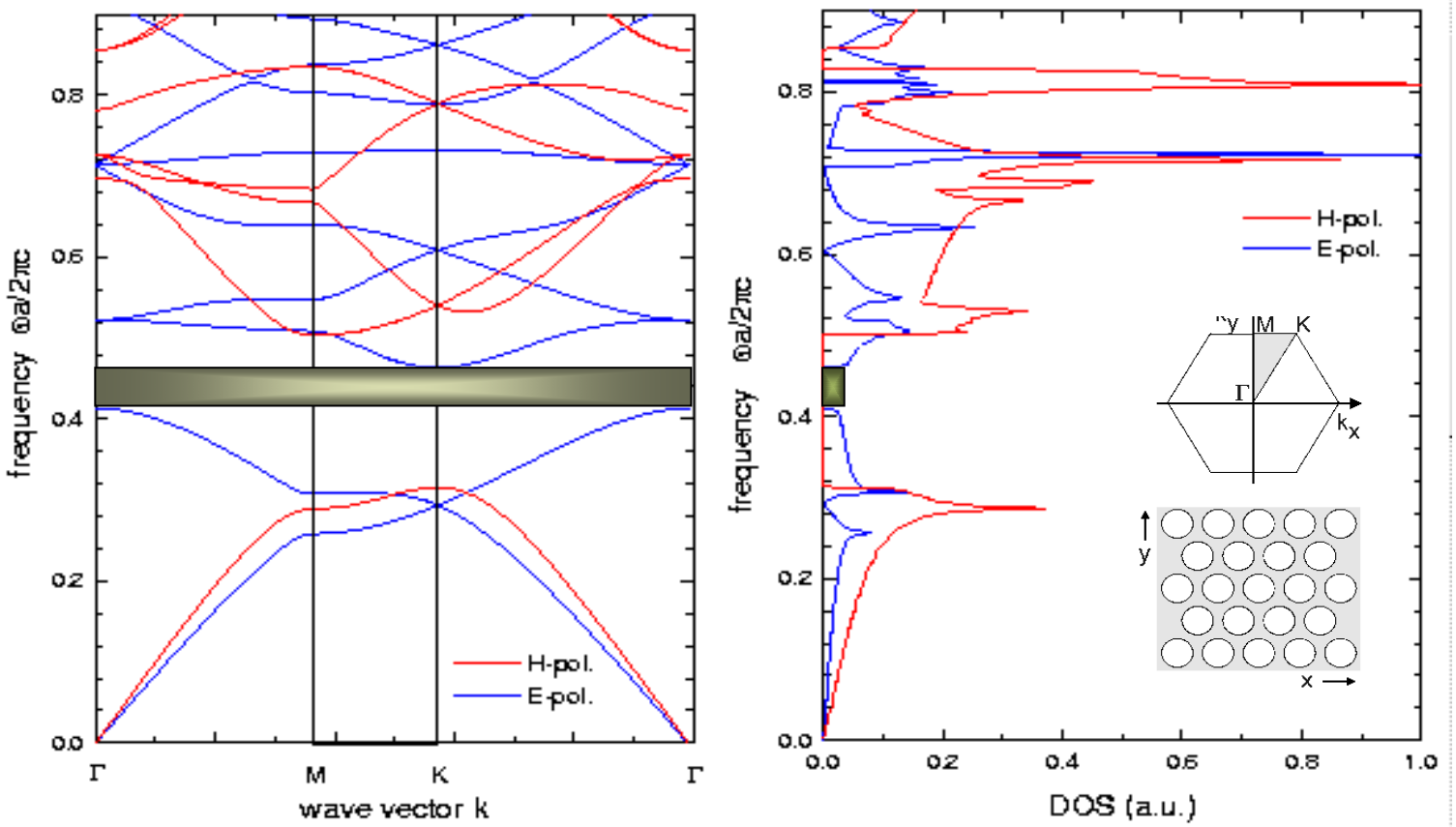
photonic band gaps (PBG) , photonic insulators



Photonic Crystals

Yablonovitch and John 1987

PBG as Normal Photonic Insulator



For electronic and photonic NI, we have mapping:

Schroedinger Eq. ↔ Maxwell's Eq. + photonic crytals

Band inversion transition in electronic systems: theoretical description



Dirac Equation (1928)

$$H = c\mathbf{p}\cdot\alpha + mc^2\beta$$

α_i and β - Dirac matrices, for example:

d=1: $\alpha_x = \sigma_x, \beta = \sigma_z.$ d=2: $\alpha_x = \sigma_x, \alpha_y = \sigma_y, \beta = \sigma_z.$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

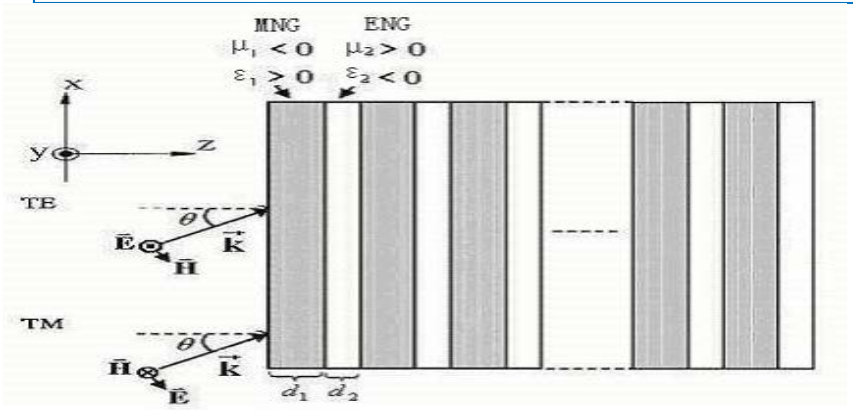
Q2: Dirac Eq. ↔ Maxwell's Eq. + artificial structures??

we proposed: Metamaterials !

2. Topological Description: theoretical study

Earlier studies: Band inversion : metematerial analogs

- PBG in a 1D stack of ε-negative (ENG)/ μ-negative (MNG) pairs
  (Jiang et al., PRE 2004, 2006; Weng et al., PRE 2007; Jiang et al. AIP Adv. 2012,)



$$\varepsilon_1 = \varepsilon_{10}, \quad \mu_1(\omega) = \mu_{10} - \frac{\alpha}{\omega^2}$$ MNG for $\omega < \alpha^{1/2}$

$$\varepsilon_2(\omega) = \varepsilon_{20} - \frac{\beta}{\omega^2}, \quad \mu_2 = \mu_{20}$$ ENG for $\omega < \beta^{1/2}$

A structure made of opaque or "dark" metamaterials!!

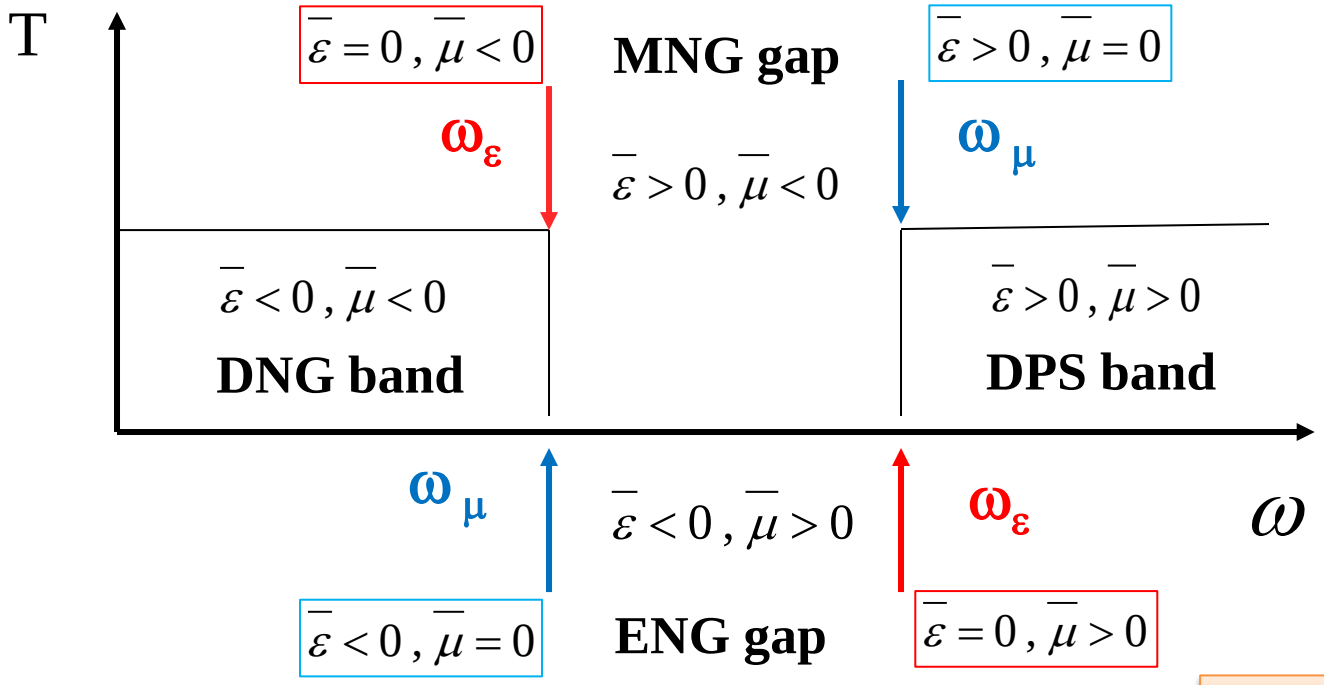
at sub-wavelength condition
and normal incidence,
a PBG structure:
with the edge ω_ε and ω_μ :

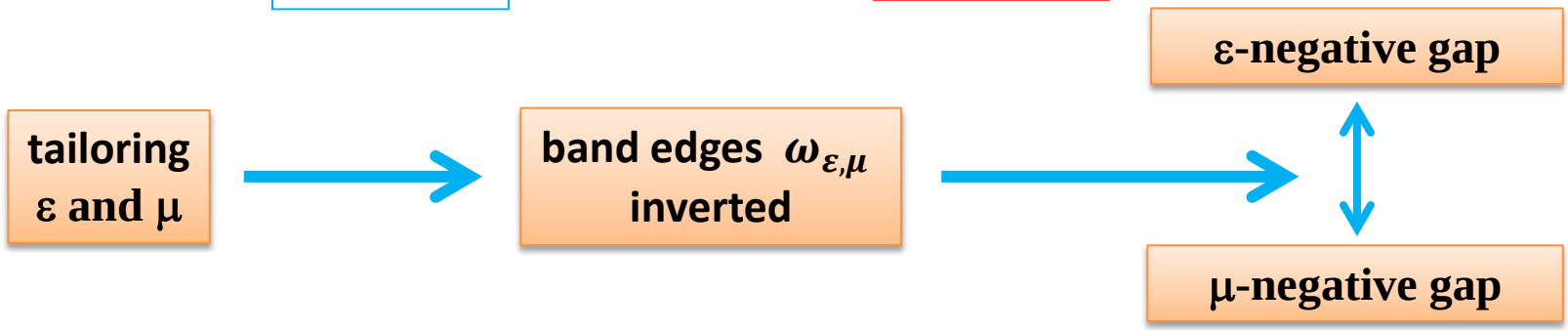
$$\overline{\varepsilon}(\omega_\varepsilon) = \frac{\varepsilon_1(\omega_\varepsilon)d_1 + \varepsilon_2(\omega_\varepsilon)d_2}{d_1 + d_2} = 0,$$

$$\overline{\mu}(\omega_\mu) = \frac{\mu_1(\omega_\mu)d_1 + \mu_2(\omega_\mu)d_2}{d_1 + d_2} = 0$$

Earlier studies:

Band inversion transition: metematerial analogs





Evidence: photonic band inversion in metamaterials

recent studies:

Photonic simulation of topological excitations in metamaterials

Wei Tan[1,2*], Yong Sun[1*], Hong Chen[1] & Shun-Qing Shen[3]

SCIENTIFIC REPORTS 2013

Answers to the two questions

1. Mapping 1D Maxwell's equations to 1D Dirac equation

Maxwell's Eq.:

$$\begin{cases} -\partial_x E_z = i\omega\mu_0\mu_r(x)H_y \\ \partial_x H_y = -i\omega\varepsilon_0\varepsilon_r(x)E_z \end{cases}$$

metamaterials

massive Dirac Eq.:

$$\left[-i\boldsymbol{\sigma}_x\partial_x + m(x)\boldsymbol{\sigma}_z + V(x)\right]\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix} = E\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}$$

$$\varphi_1 = \sqrt{\varepsilon_0}E_z \quad \varphi_2 = \sqrt{\mu_0}H_y \qquad E = -\frac{\omega}{2c}\langle\varepsilon_r + \mu_r\rangle$$

$$m(x) = \frac{\omega}{2c}(\varepsilon_r - \mu_r) \qquad V(x) = \frac{\omega}{2c}\left[(\varepsilon_r + \mu_r) - \langle\varepsilon_r + \mu_r\rangle\right]$$

<.....> : Average on space

2. EMG and MNG mirror as mass inversion in Dirac Eq.

For SNG mirror, if :

$$\epsilon_r \sim -\ \mu_r\ , E = \frac{\omega}{2c}\langle \epsilon_r + \mu_r \rangle \sim 0$$

Then at low energy $E \sim 0$:
the behavior of Dirac Eq. ONLY depends on the sign of the mass

$$m = \frac{\omega}{2c}\langle \epsilon_r - \mu_r \rangle \sim \frac{\omega}{c}\ \varepsilon_r \sim -\frac{\omega}{c}\mu_r$$

So, the sign of the mass is inverted from MNG to ENG :

MNG mirror: $\varepsilon_r > 0.\ \mu_r < 0$ ⟷ positive mass: m > 0

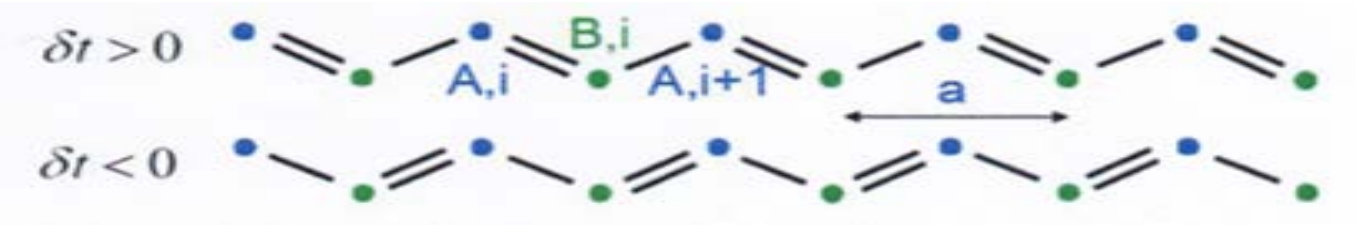
ENG mirror: $\varepsilon_r < 0.\ \mu_r > 0$ ⟷ negative mass: m < 0

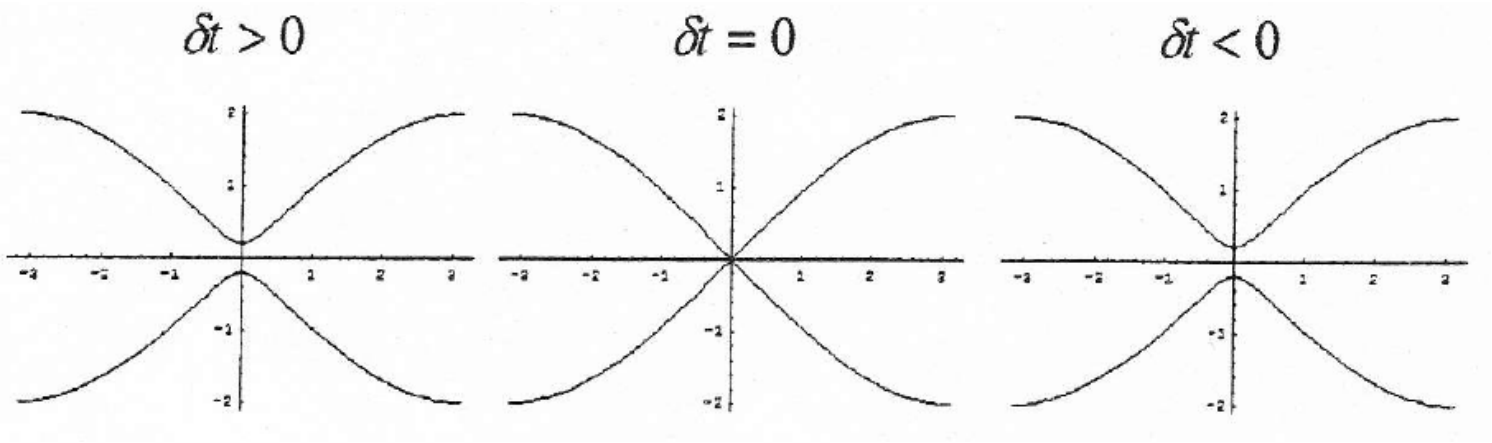
Different topological orders for MNG and ENG: The first evidence

Mapping the Dirac Eq. to the SSH model (S.Q Shen 2013)

Su-Schrieffer-Heeger Model for Polyacetylene (Rev. Mod. Phys. 1988)

$$H = \sum_{n=1}^{N}(t+\delta t)c_{A,n}^{\dagger}c_{B,n} + \sum_{n=1}^{N-1}(t-\delta t)c_{A,n+1}^{\dagger}c_{B,n} + h.c.,$$





$m > 0$ $m = 0$ $m < 0$

Band inversion in the SSH model ⟷ Mass inversion in the Dirac Eq.

Berry phase:

$$\gamma = \int_{-\pi}^{+\pi} dk \, \langle \varphi | \, i \, \partial_k \, | \varphi \rangle = \frac{1}{2}\pi \, [\mathrm{sgn}(t + \delta t) - \mathrm{sgn}(\delta t)]$$

$$= \begin{cases} 0 \;\; for \; \Delta t > 0 \\ \pi \; for \; \Delta t < 0 \end{cases}$$

winding number:

$$(-1)^{\nu} = \mathrm{sgn}(\delta t)\mathrm{sgn}(t + \delta t) = \mathrm{sgn}(1 + t/\delta t)$$

$$\nu = \begin{cases} 0 \; for \;\; \Delta t > 0 \; or \; m > 0 \text{ and } \textbf{MNG mirror} \\ 1 \; for \; \Delta t < 0 \; or \; m < 0 \text{ and } \textbf{ENG mirror} \end{cases}$$

Therefore, MNG and ENG mirrors have Different topological orders!

Topological description:
Extend to mirrors made of dielectric multilayers

Our study: Guo et al., PRE 2008; Chin. Phys. B 2008
multilayer structures or1D-PC
made of dielectrics with ($\varepsilon > 1$, $\mu = 1$)
can act as SNG metamaterials in gap region

Depending on symmetry of the unit cell.

For asymmetry unit cell: $(AB)_m$

The gap divided into two parts: EMG and MNG

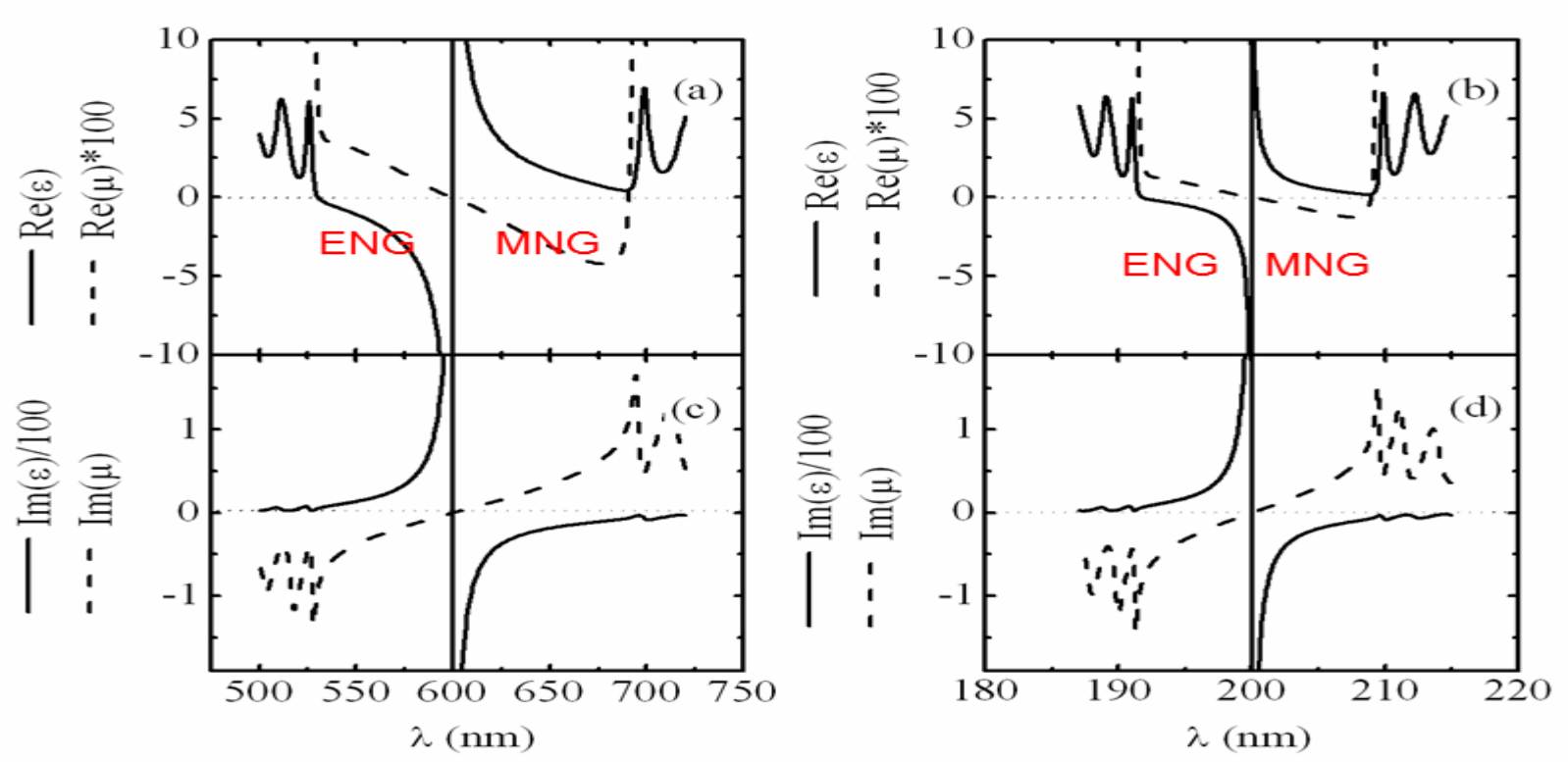
For symmetry unit cell: $(ABA)_m$

The gap described by: EMG or MNG

Determination of effective parameters

- Retrieval theory : Smith et al., *PRB 2002*
- Bloch-wave-expansion theory: Kan et al., PRA 2009

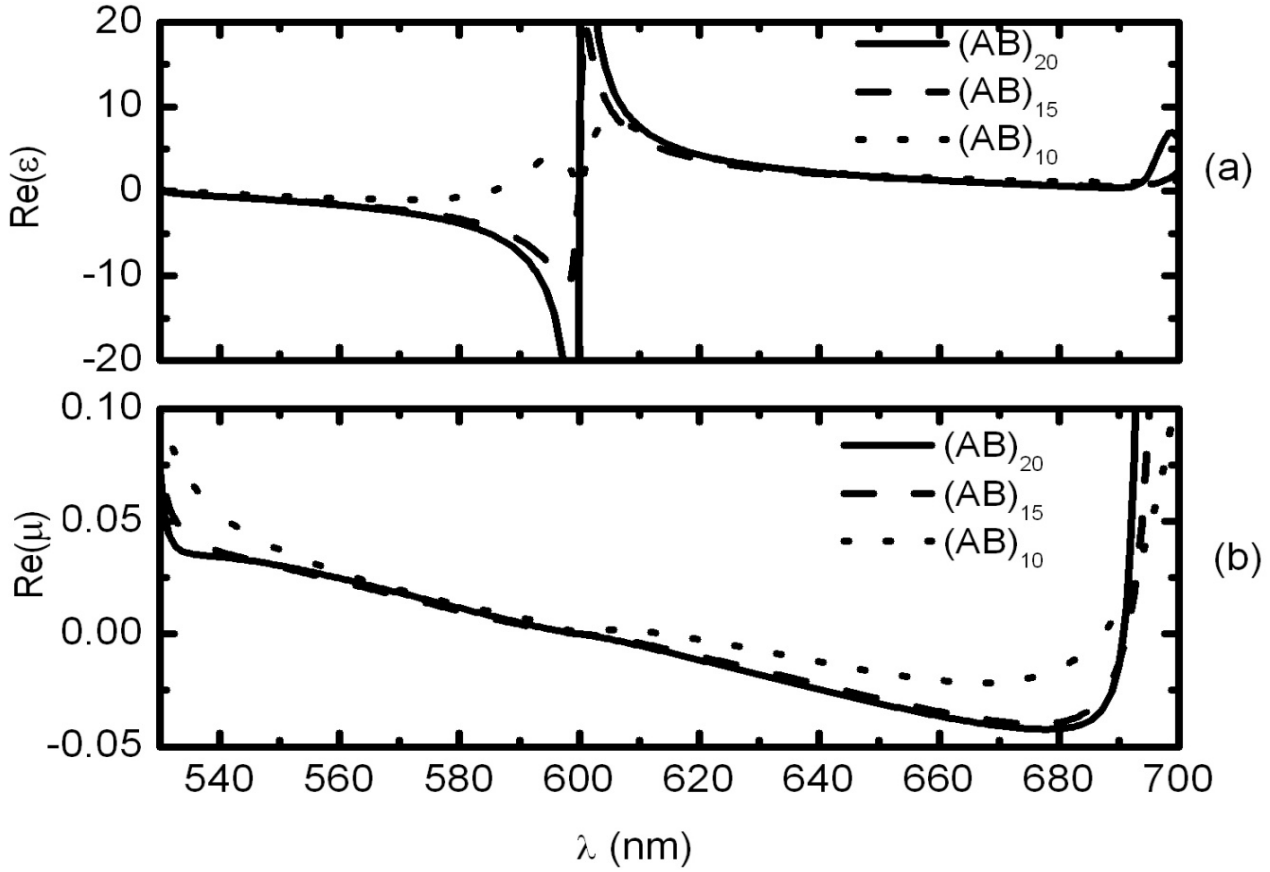
Effective parameters in gap regime $(AB)_{20}$



The gap is divided into two parts: EMG and MNG

Dependence on periodic number

$(AB)_m$: $m = 10, 15, 20$



non-local effective parameters !!

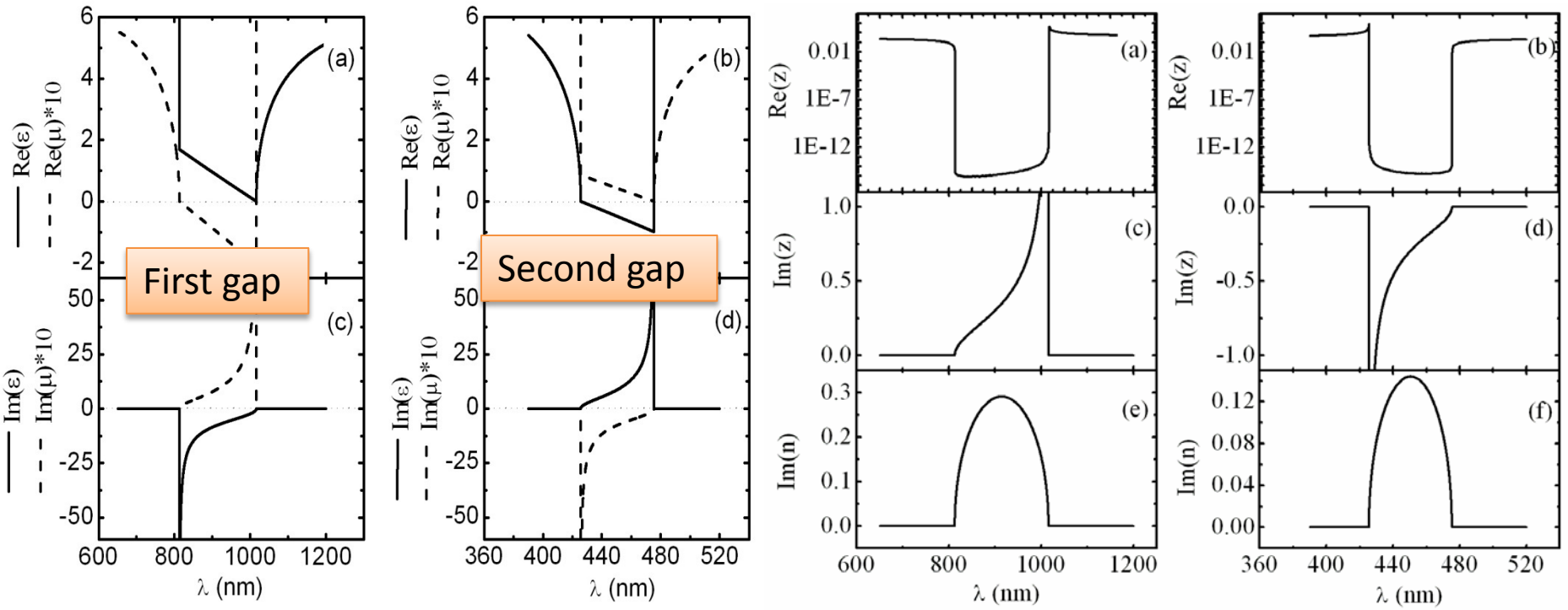
For asymmetry unit cell: $(AB)_m$

Effective parameters in gap regime

$(ABA)_{10}$

MNG

ENG



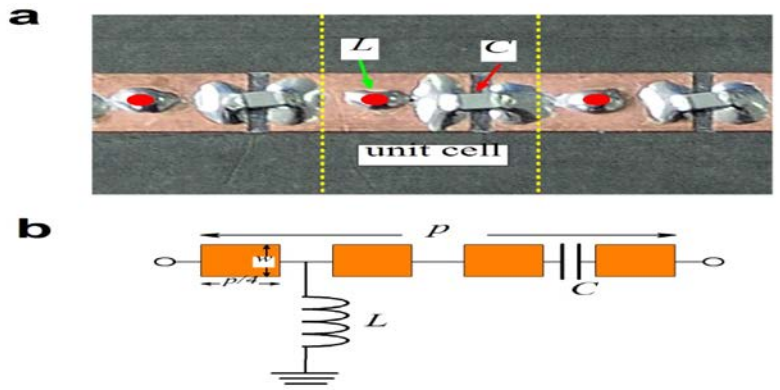
It can be shown:

E and μ are independent of the periodic number

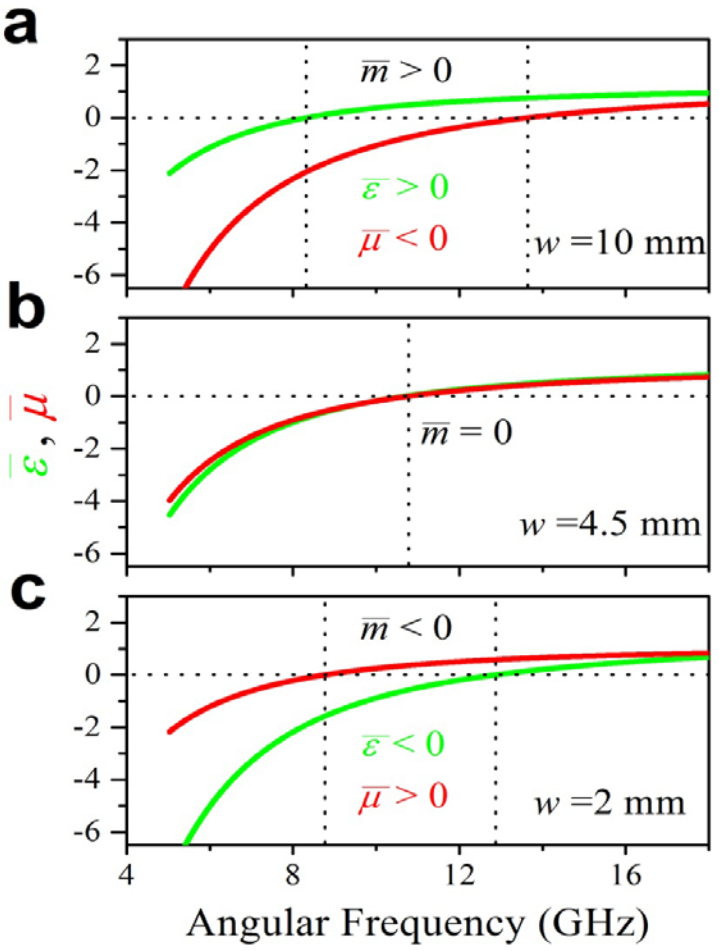
local effective parameters

3. Topological Description: experimental study

1D Metamaterials Realized By Transmission Line



Eleftheriade et al., 2002; and by Itol et al., 2002

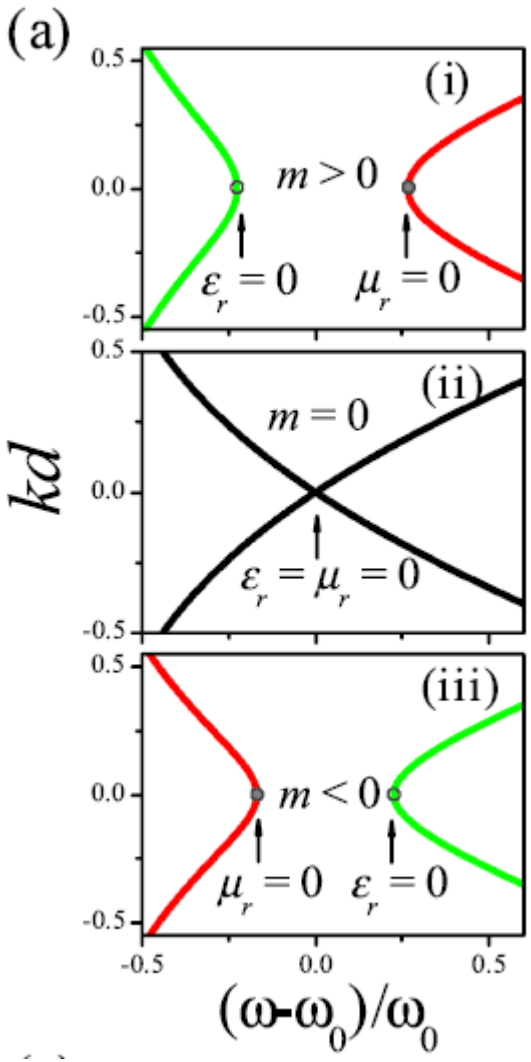


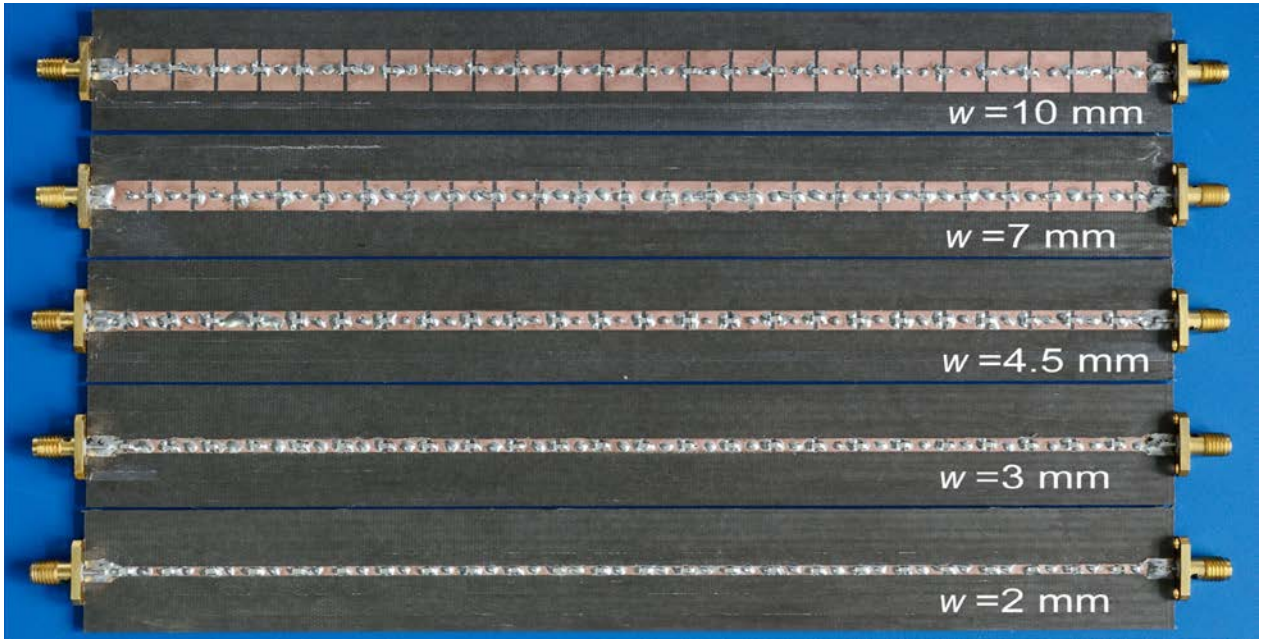
$$\overline{\varepsilon}_r = \frac{1}{p\varepsilon_s}\left(C_0 - \frac{1}{\omega^2 L d}\right) + i\frac{\gamma_e}{\omega} \qquad \overline{\mu}_r = \frac{p}{\mu_s}\left(L_0 - \frac{1}{\omega^2 C d}\right) + i\frac{\gamma_m}{\omega}$$

choosing different circuit parameters one gets DNG, ENG, MEG materials

$$m(x) = \frac{\omega}{2c}\left(\varepsilon_r - \mu_r\right)$$

Band Inversion in photonic chains

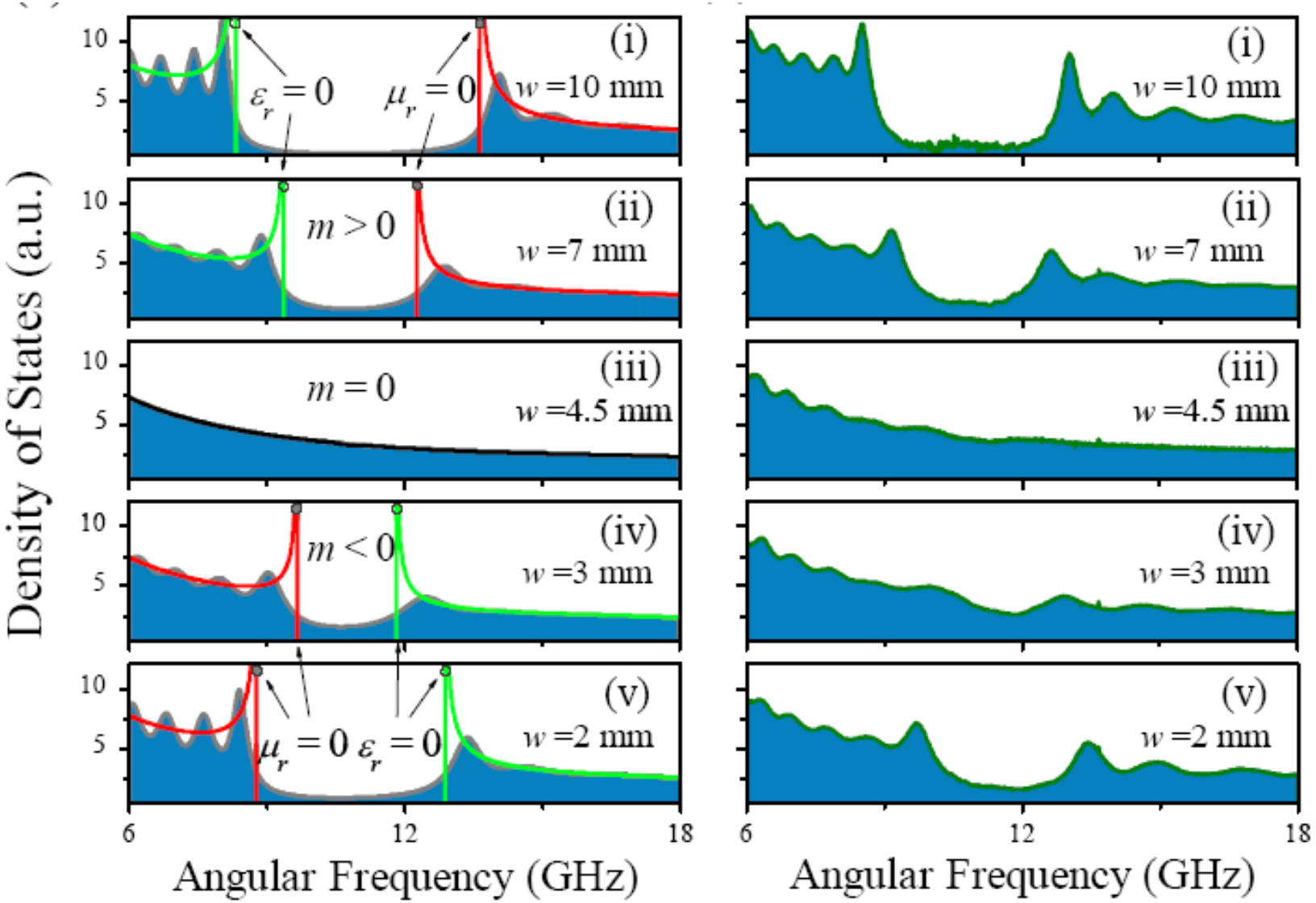




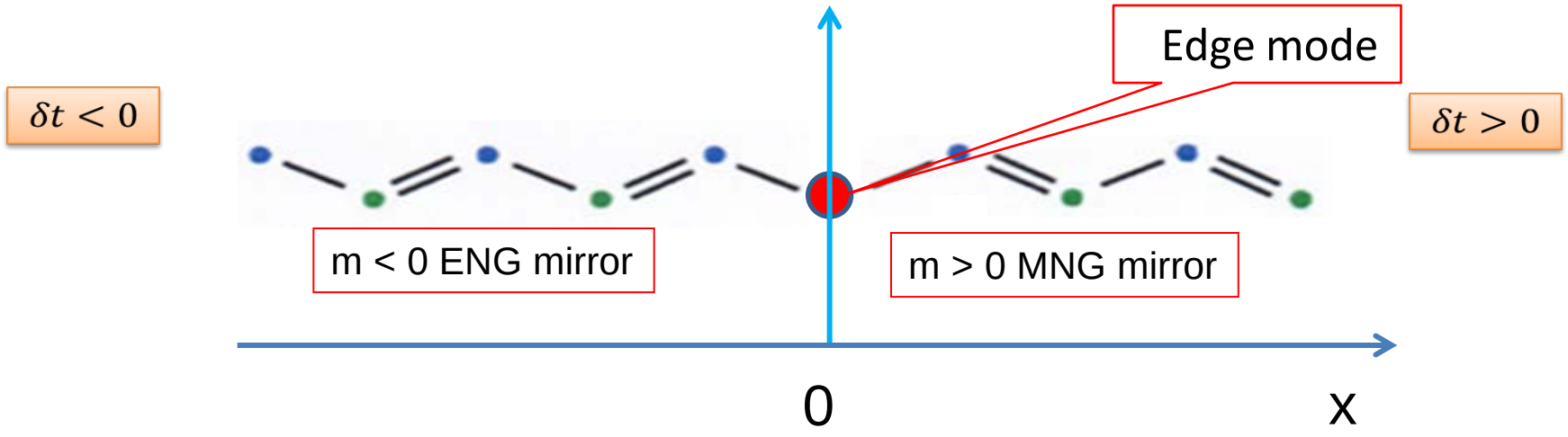
DOS: $g(\omega) = \frac{1}{\pi}\frac{dk}{d\omega} \propto \frac{\tau_g}{\pi D}$

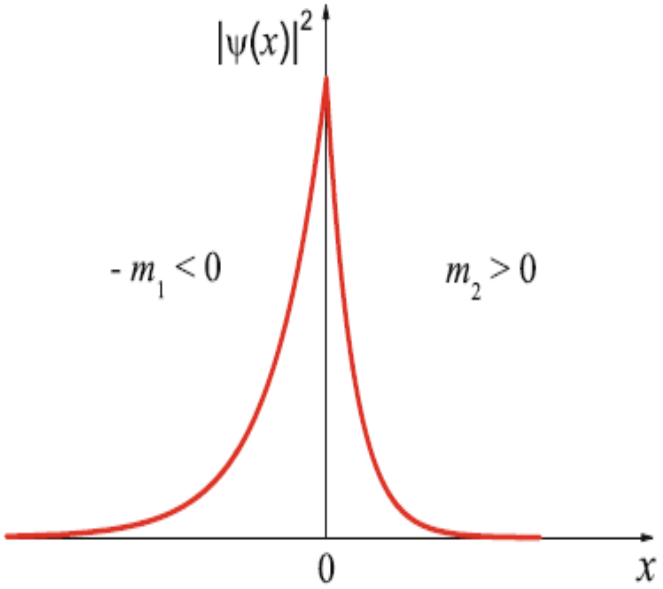
τ_g : group delay D : sample length

Simulations & Experiments



Edge modes in heterostructures made of ENG and MNG: Theory prediction





Jackiw-Rebbi Solution (Phys. Rev. D 1976)

Edge mode:

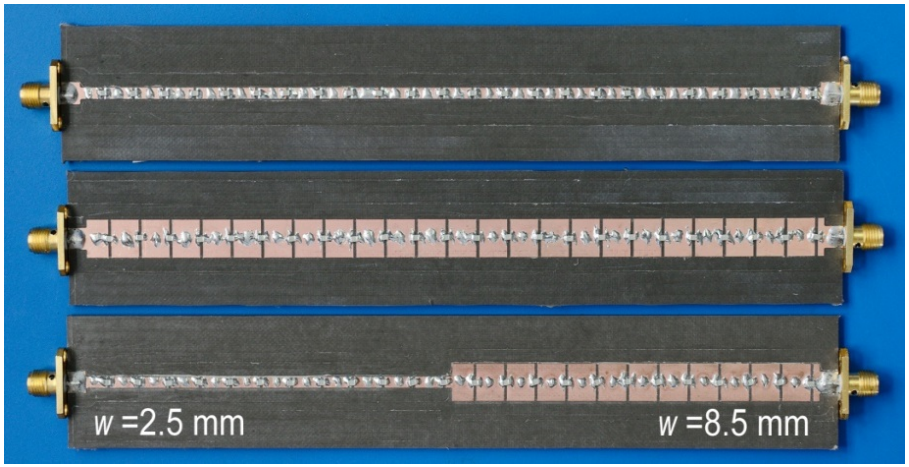
$$\varphi_\eta(x) \propto \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \eta i \end{pmatrix} \exp\left[- \int^x \eta \frac{m(x')v}{\hbar} dx' \right].$$

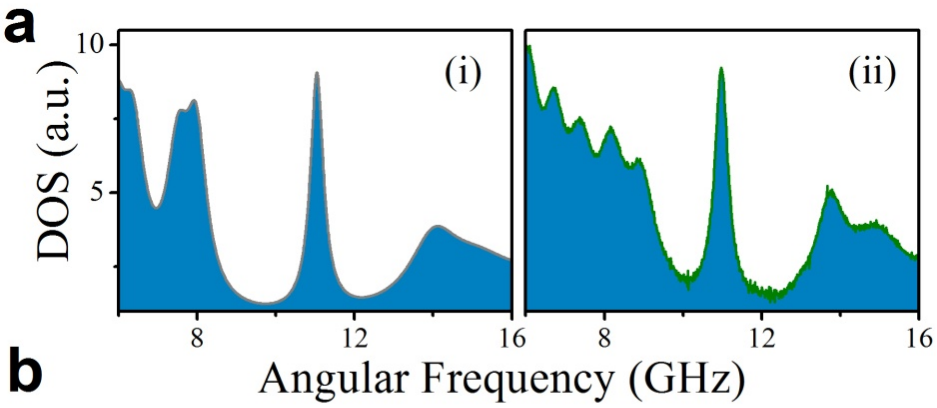
for $$m(x) = \begin{cases} -m_1 & \text{if} \quad x < 0 \\ +m_2 & \text{otherwise} \end{cases}$$

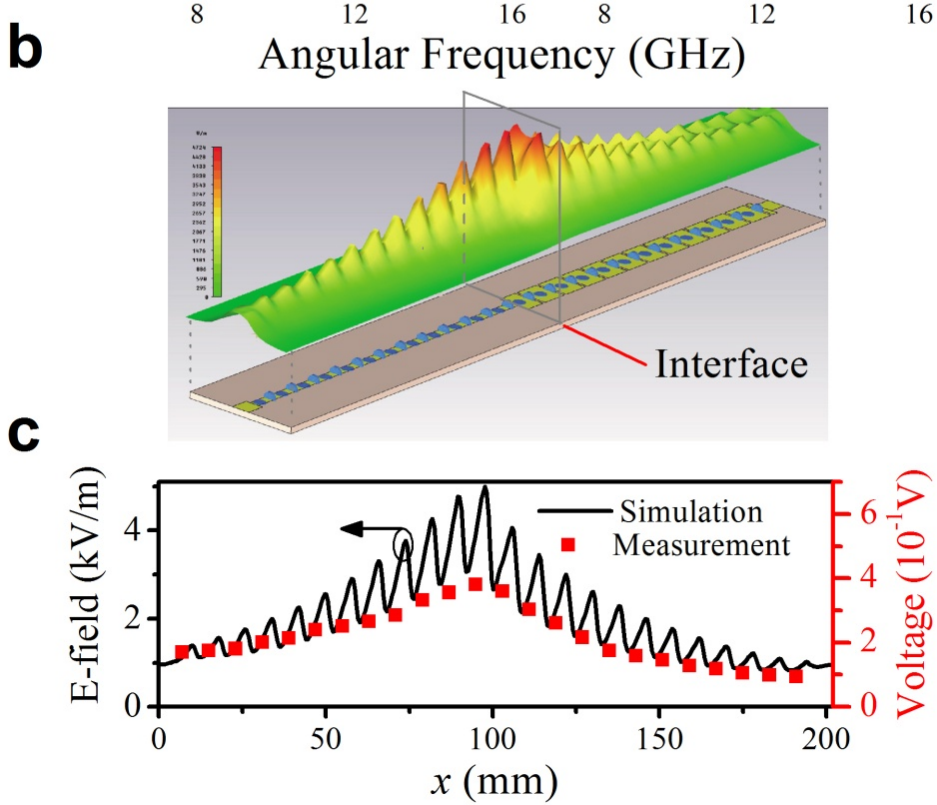
$$\Psi(x) = \sqrt{\frac{v}{\hbar} \frac{m_1 m_2}{m_1 + m_2}} \begin{pmatrix} 1 \\ i \end{pmatrix} e^{-|m(x)vx|/\hbar}$$

Edge modes in heterostructures made of ENG and MNG: Microwave experiments

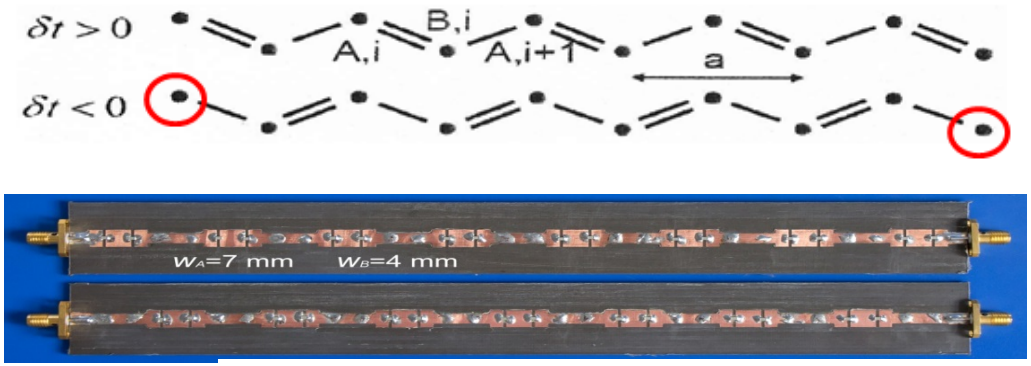
Edge mode at the interface between two photonic mirrors with $m>0$ and $m<0$

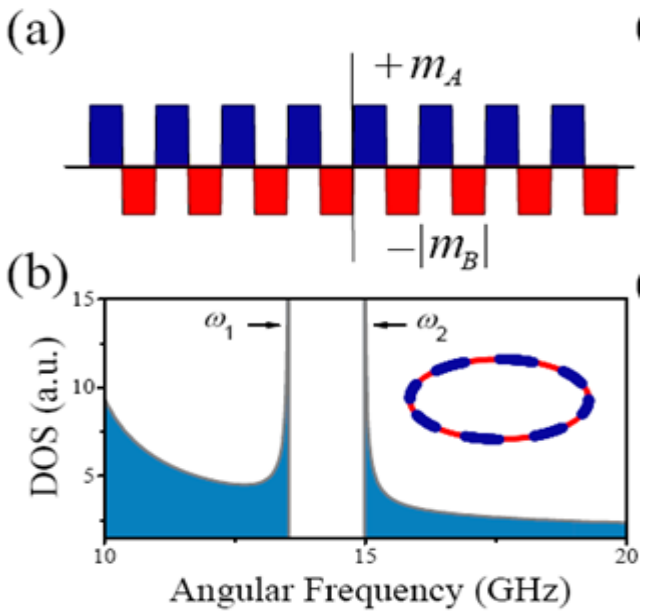






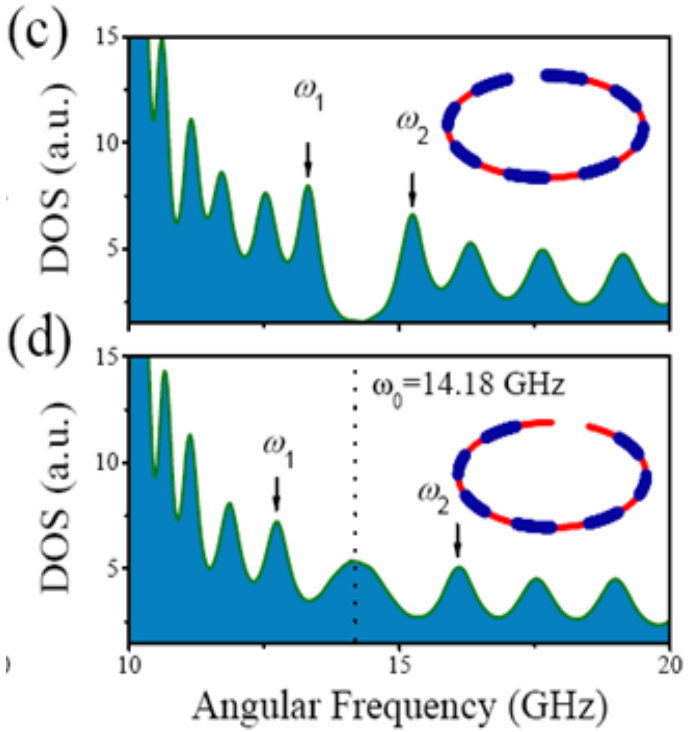
Edge modes in photonic chains: Microwave experiments

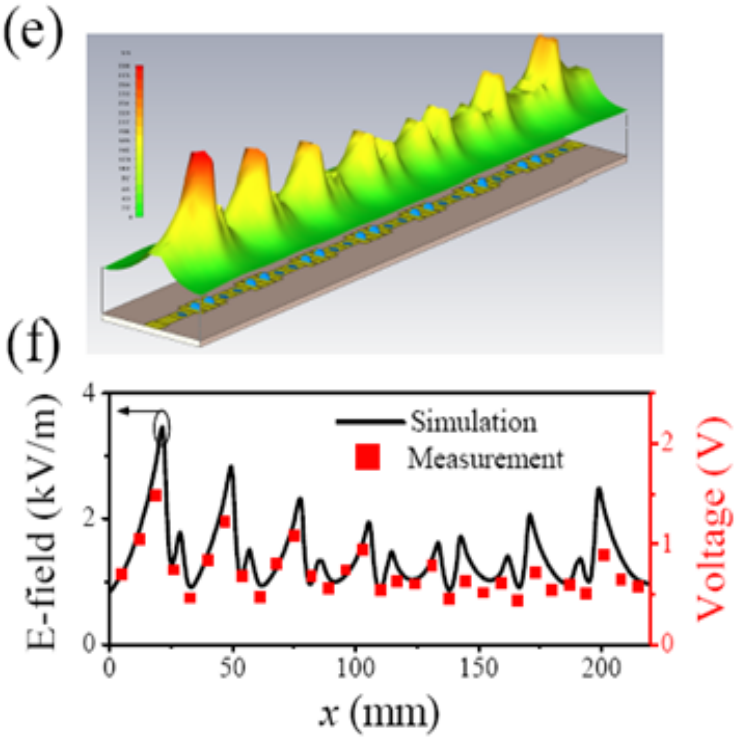




$\delta t > 0$

$\delta t < 0$

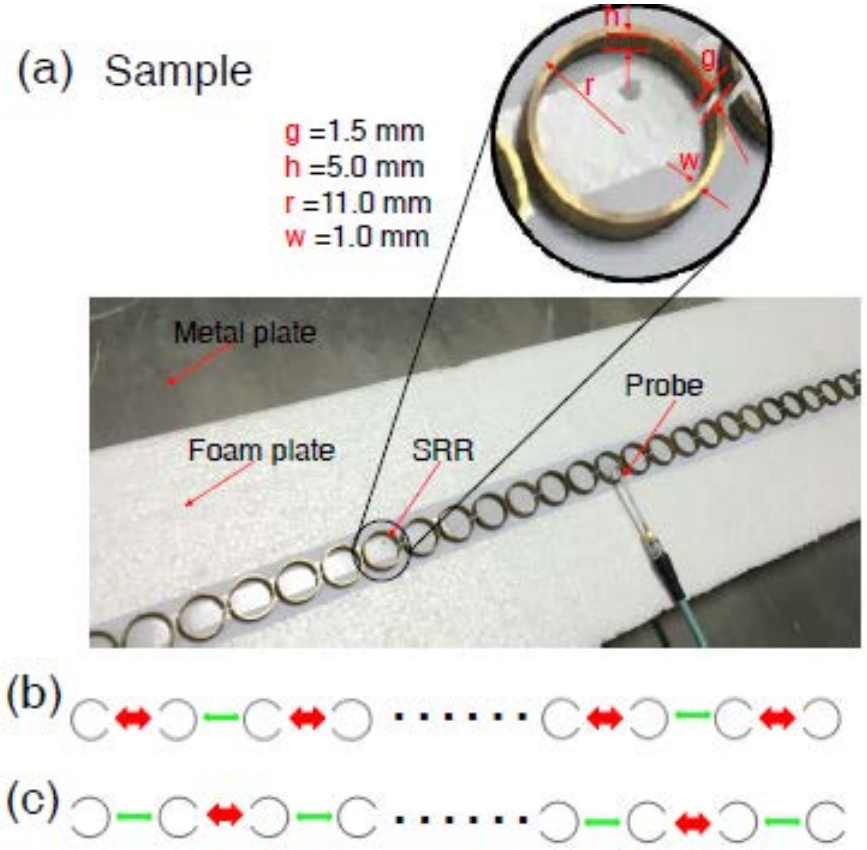


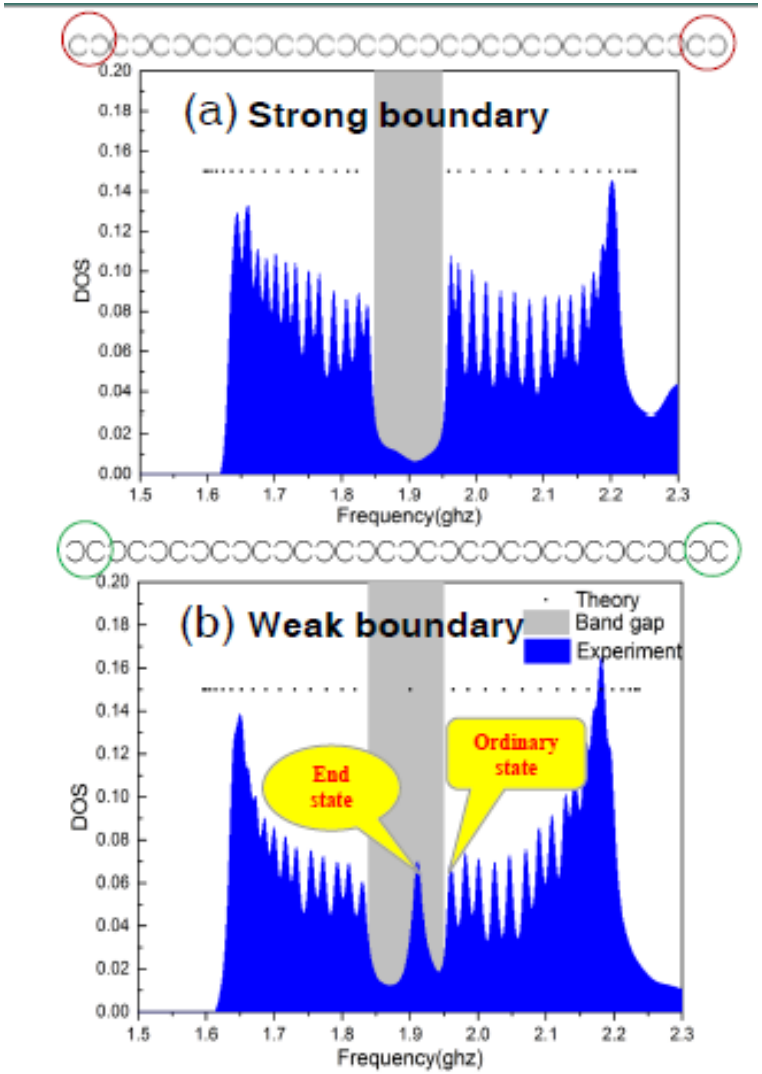


Robust end states against loss and disorder in a photonic chain made of split-ring-resonators

Jun Jiang, Yong Sun, Fusheng Deng, Dian Yuan and Hong Chen*

MOE Key Laboratory of Advanced Micro-structure Materials, Tongji University, Shanghai 200092, China
hongchen@tongji.edu.cn

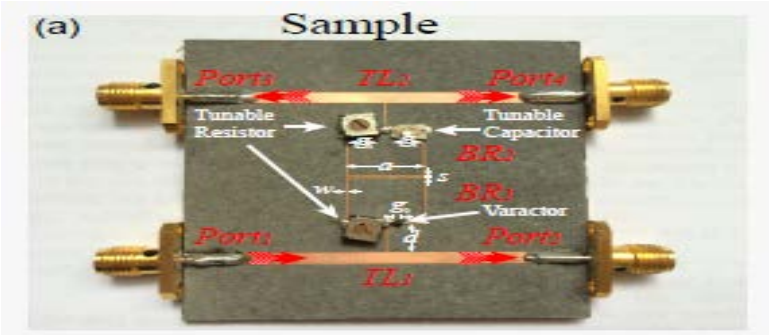


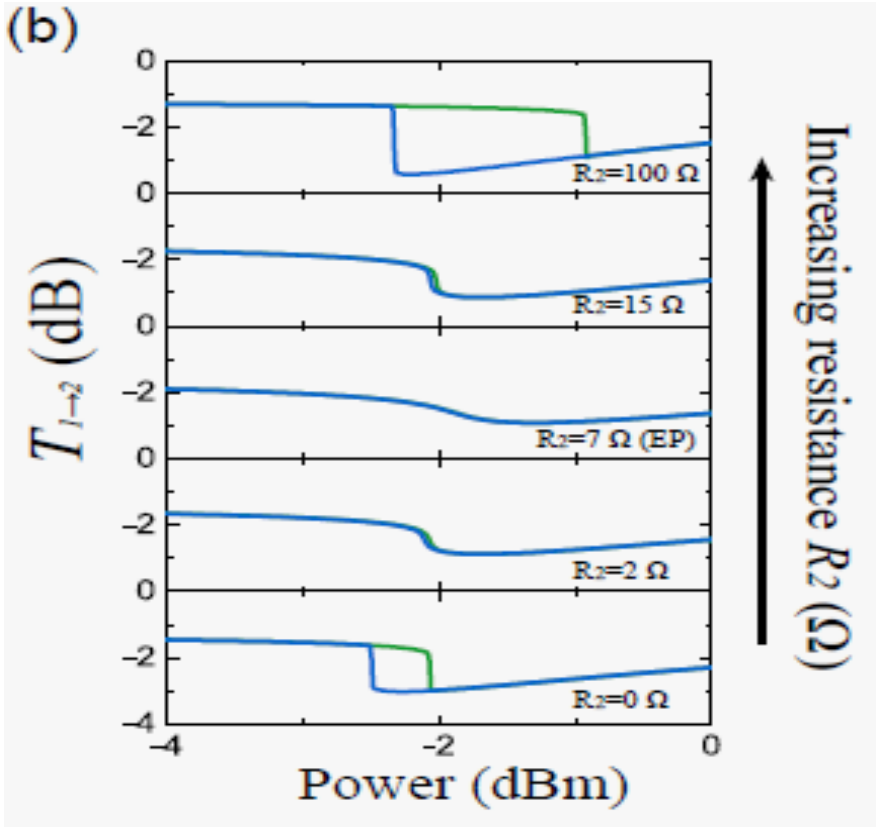


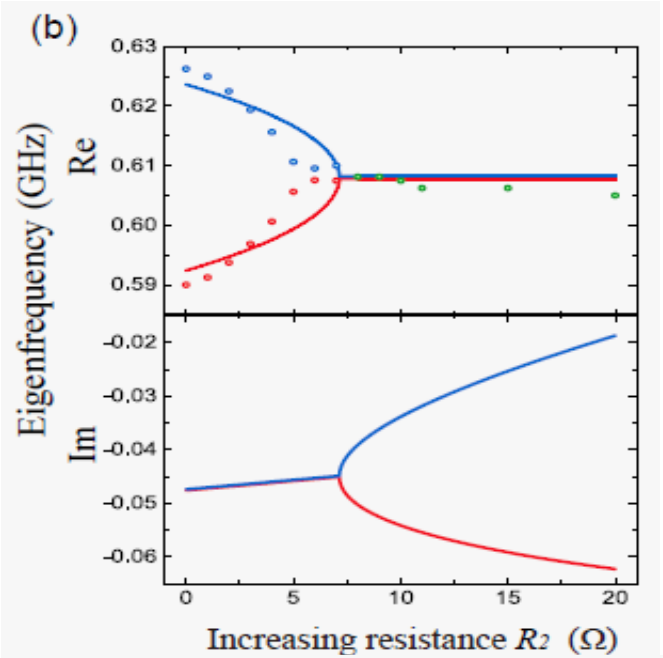
Poster presented by Jun Jiang

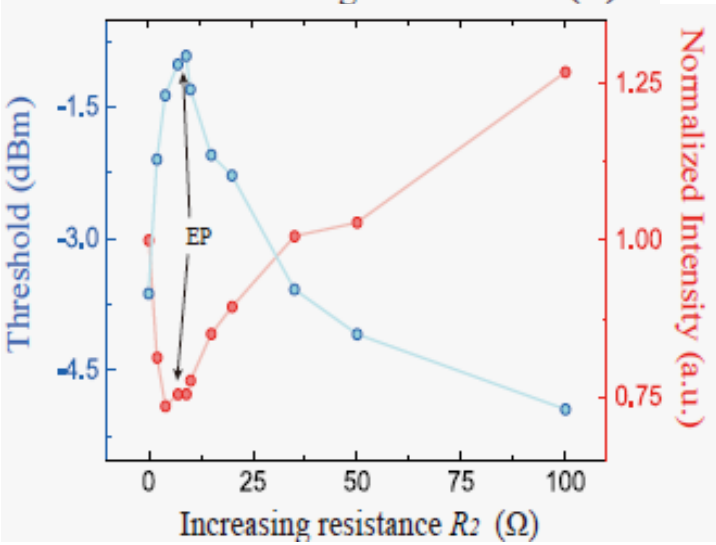
Revival of bistable state near the exceptional point of a non-Hermitian photonic molecule

Poster presented by Kejia Zhu



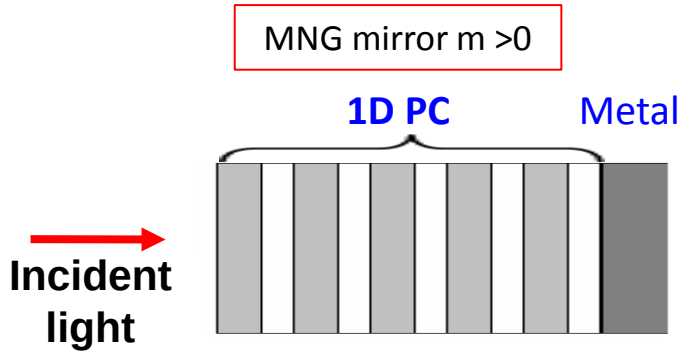




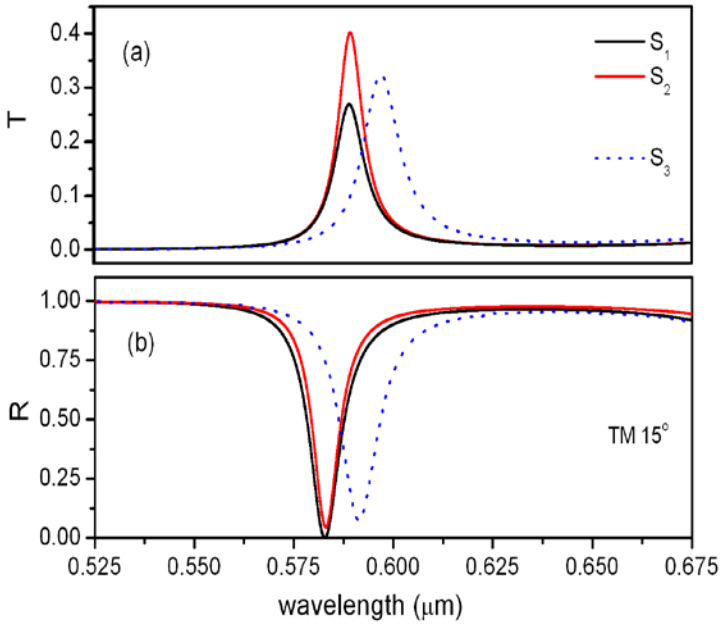


Edge mode in heterostructures made of ENG and MNG: Visible-light experiments

Edge mode in heterostructure: $(AB)_6M$



A: SiO_2
B: TiO_2
M: Ag



----- S_1 , ----- S_2
theoretical results
with different losses
$\theta = 0^o$

..... S_3
experimental results
$\theta = 15^o$

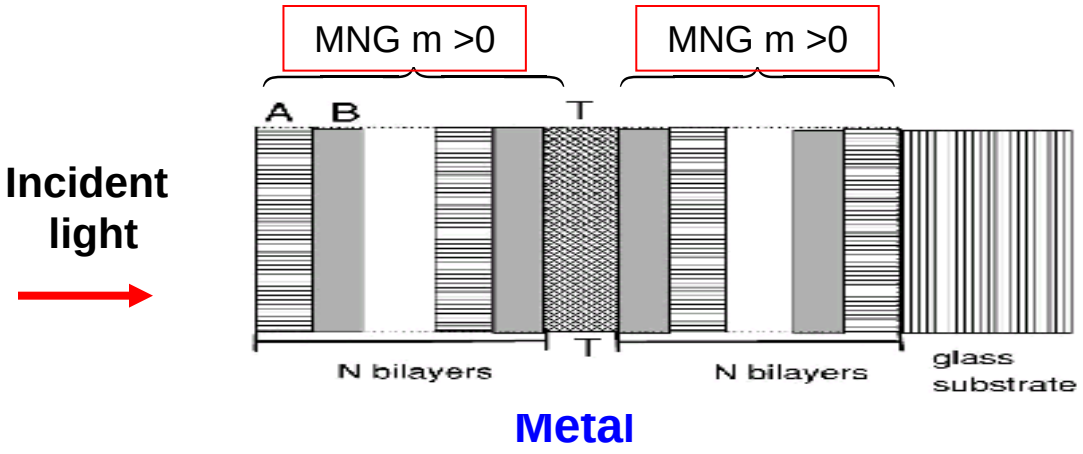
$n_A = 1.443$, $n_B = 2.327$

$d_A = 89.0\,nm$, $d_B = 55.5\,nm$

$d_M = 60.2\,nm$

For λ = 589 nm, d_M = 60.2 nm
T < 1% without edge mode
T = 33% with edge mode
Enhancement: 30 ~ 40

Edge mode in sandwich structure: $(AB)_6M(BA)_6$

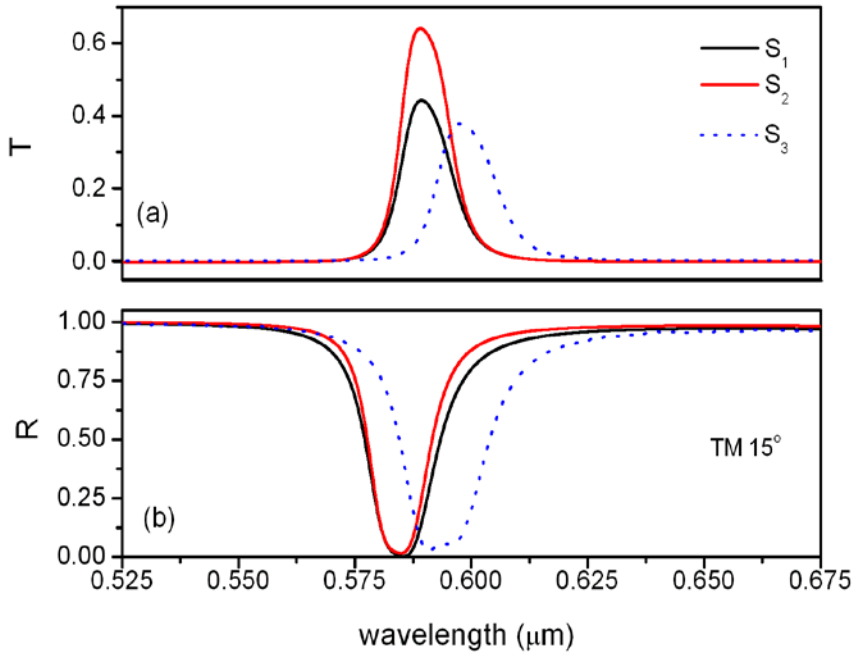


$(AB)_5M(BA)_5$ S;
A: SiO_2 ; B: TiO_2 ;
M: Ag ; S: glass

$n_A = 1.443$, $n_B = 2.327$

$d_A = 89.0\ nm$, $d_B = 55.5\ nm$

$d_M = 83.1\ nm$



----- S_1 , ----- S_2
theoretical results
with different losses
$\theta = 0^o$

..... S_3
experimental results
$\theta = 15^o$

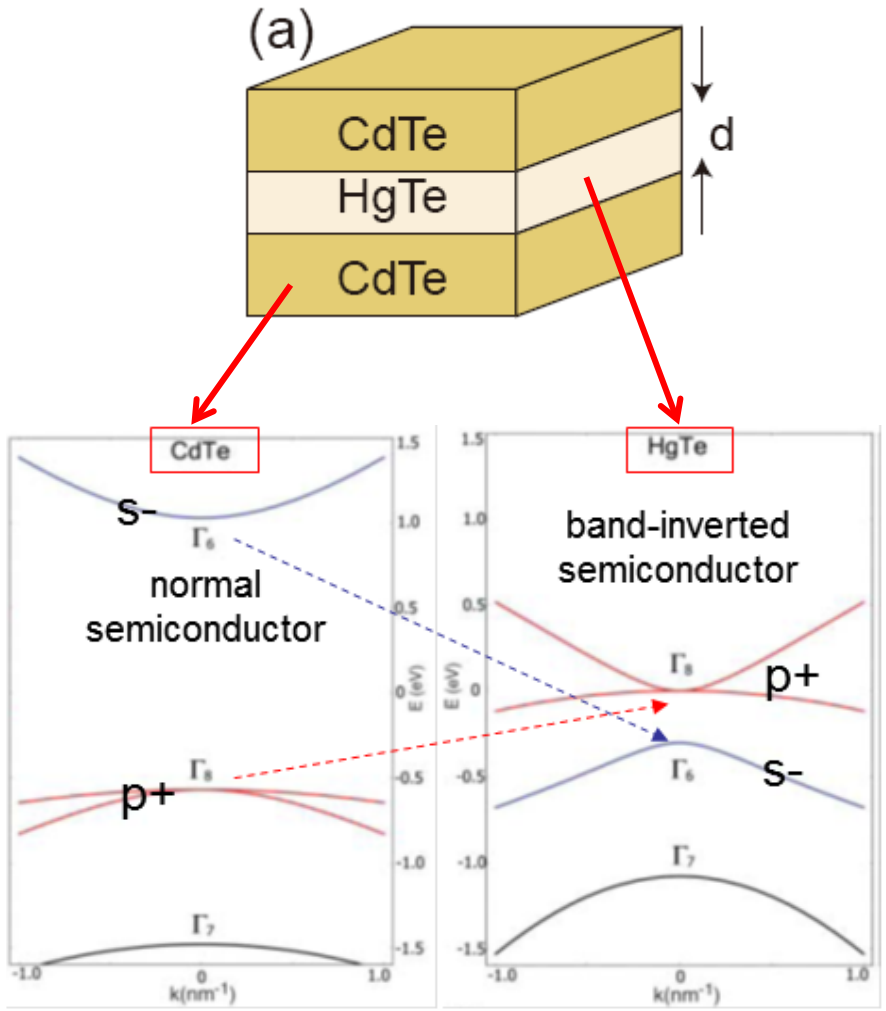
For λ = 589 nm , d_M = 83.1 nm
T = 0.15% without edge mode
T = 38% with edge mode
Enhancement: 255

- ✓ OPTICAL THICK metal film
- ✓ FAR FIELD excitation

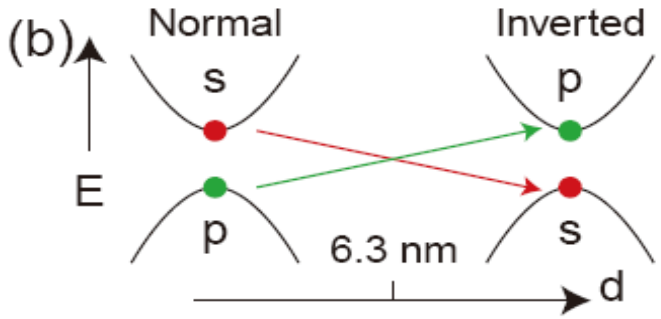
Possible applications: plasmonics

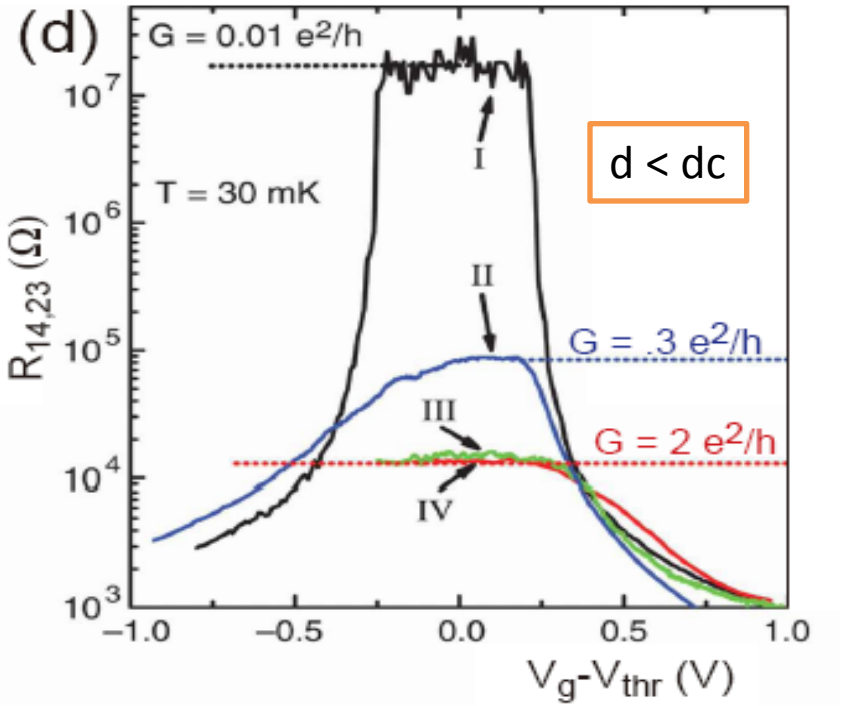
comparing to sandwich structures of electronic TI

M. Z. Hasan *et al.*, *Rev. Mod. Phys.* (2010)



NI TI

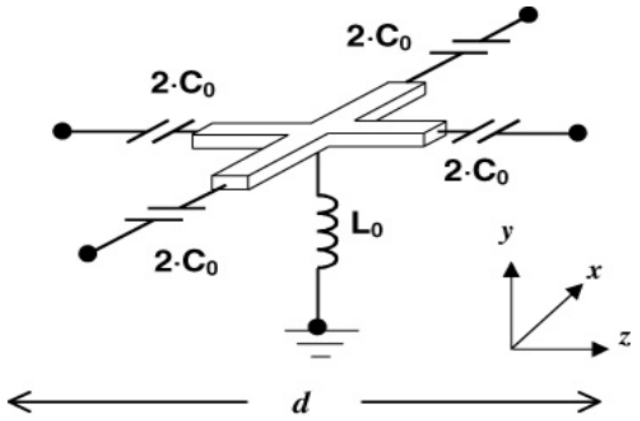




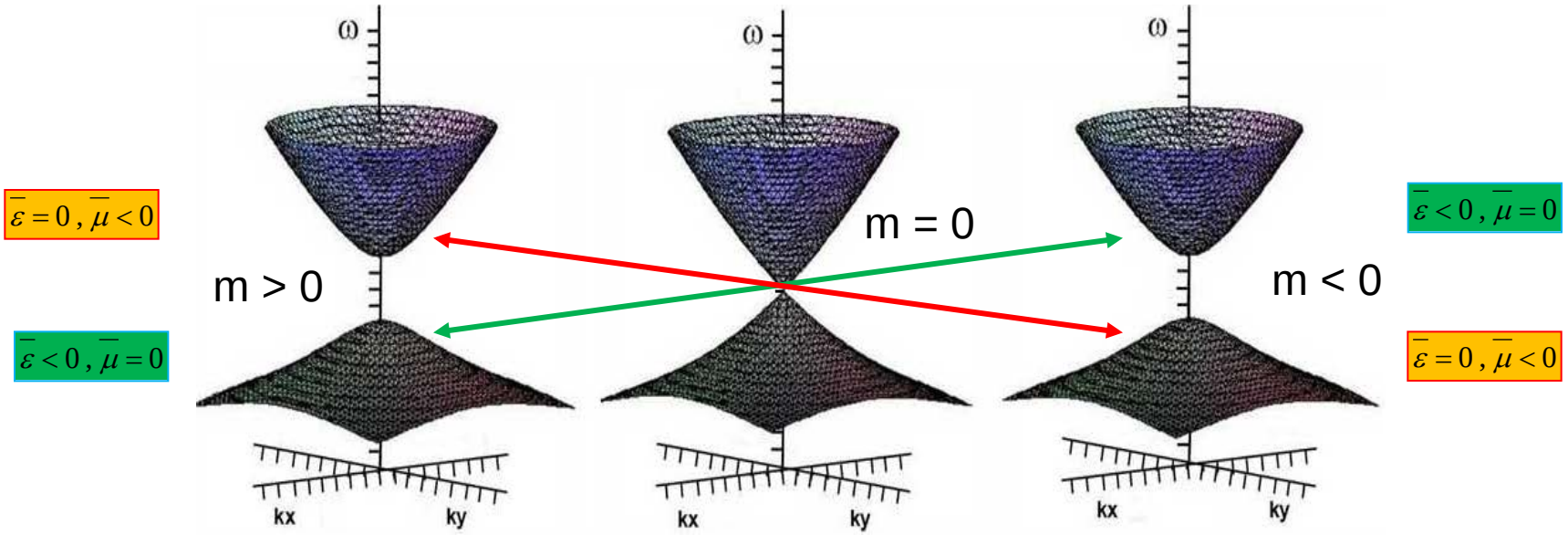
Edge state → Resisdence 10^{-2} !

Extend to 2D structures

Plannar metamaterials made of transmission lines



Band gap inversion transition

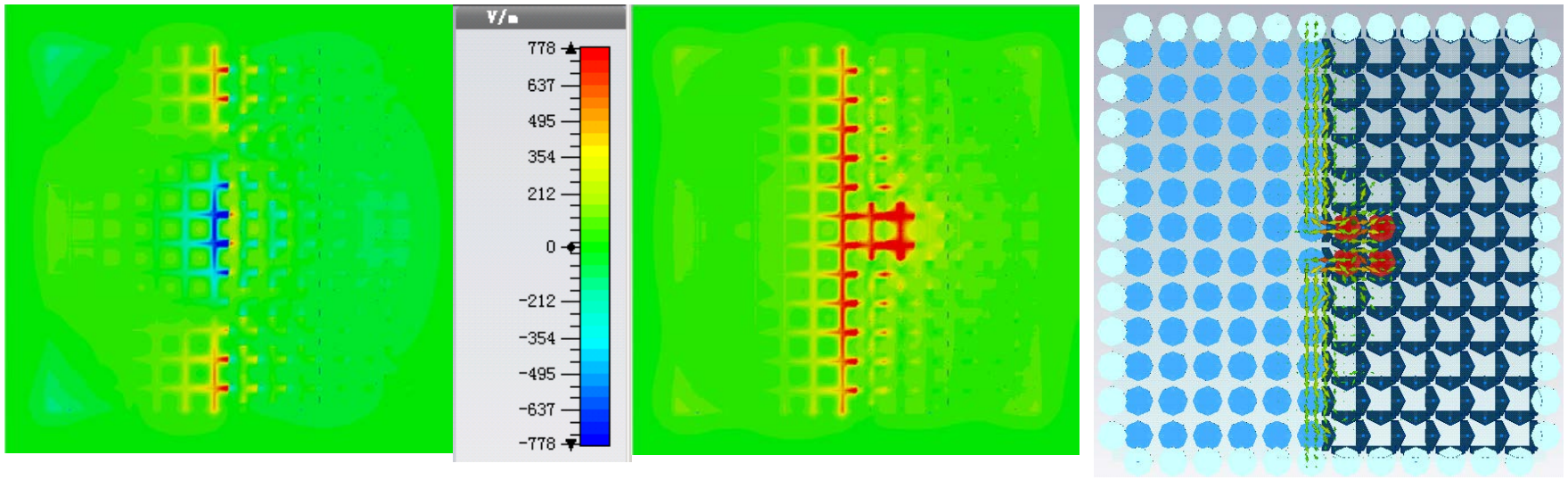


4654 OPTICS LETTERS / Vol. 37, No. 22 / November 15, 2012

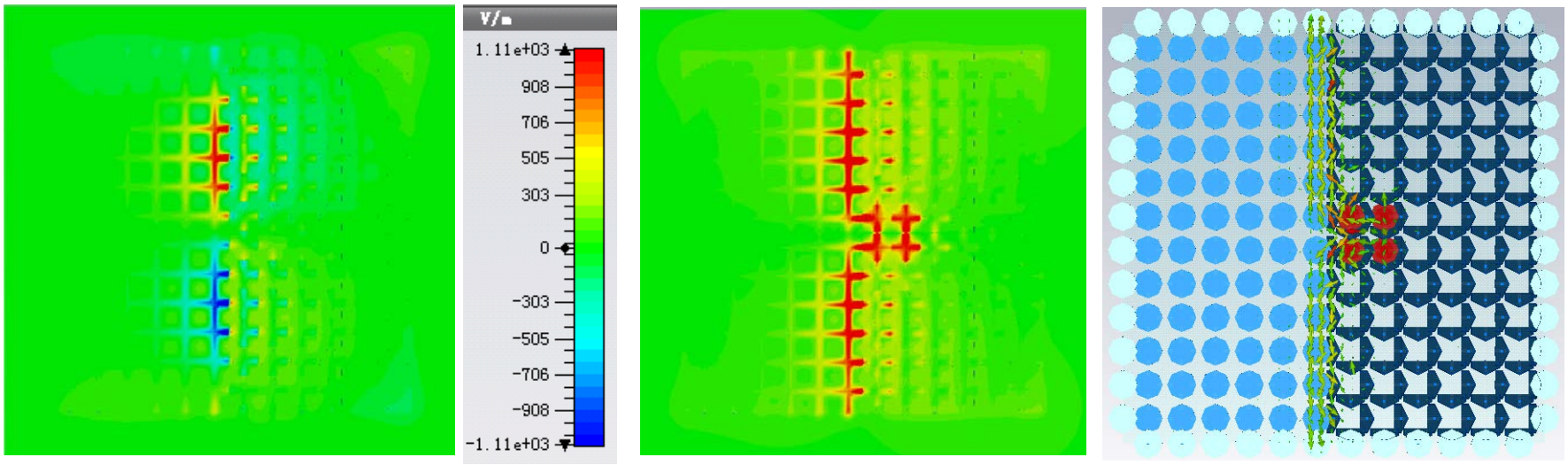
Microwave collimation based on zero index metamaterials with Dirac point

Kai Fang, Yewen Zhang,* Fangfei Li, Haitao Jiang, Yunhui Li, Wusong Wang, and Hong Chen

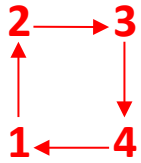
x- direction linearly polarized source

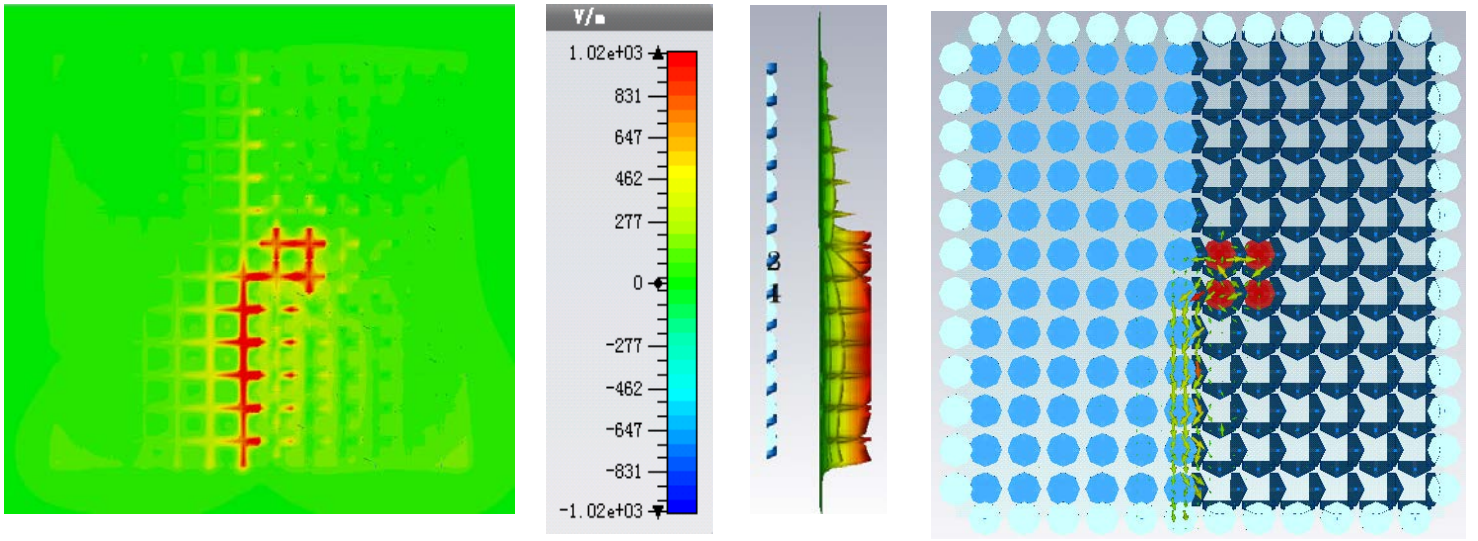


y- direction linearly polarized source

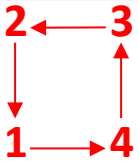


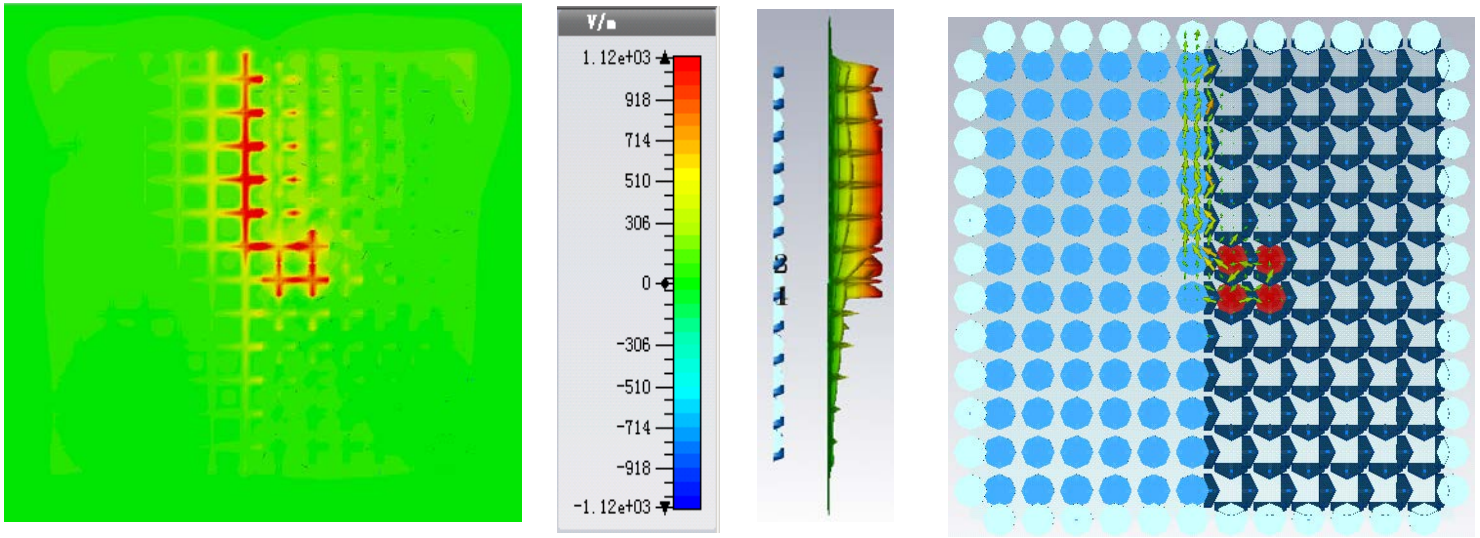
clockwise circularly polarized source





counterclockwise circularly polarized source





4. Summary

- Mapping Maxwell's equation to Dirac Eq., it is shown ϵ-negative and μ-negative mirrors have different topological orders.
- Realizing topological modes in structures made of photonic mirrors.
- Proving new ways of applications based on photonic topological modes.

Financial Supports: NSFC, 973 Program of MOST

Thank You