Metamaterials with Gain

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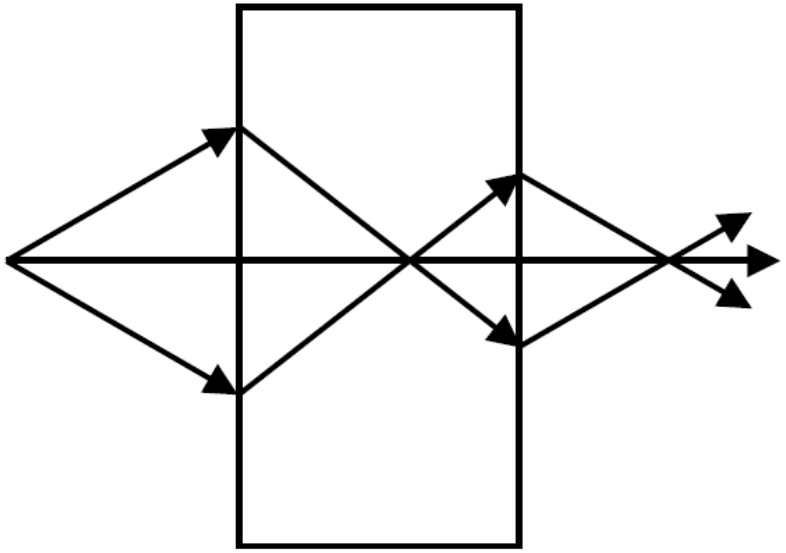
15 Jan 2016

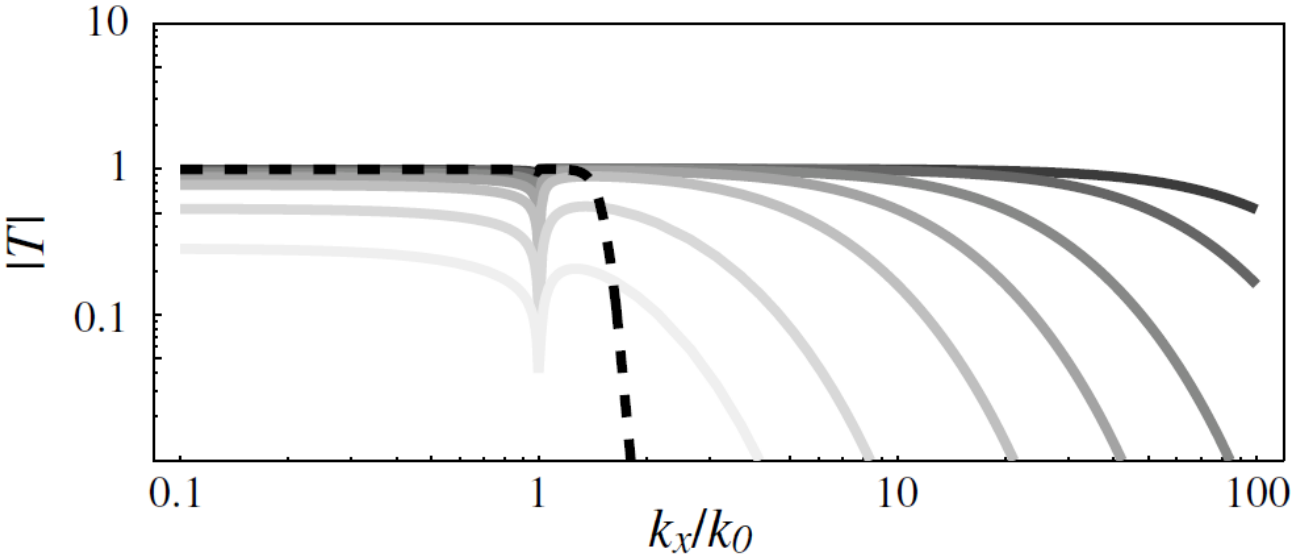
Loss problem



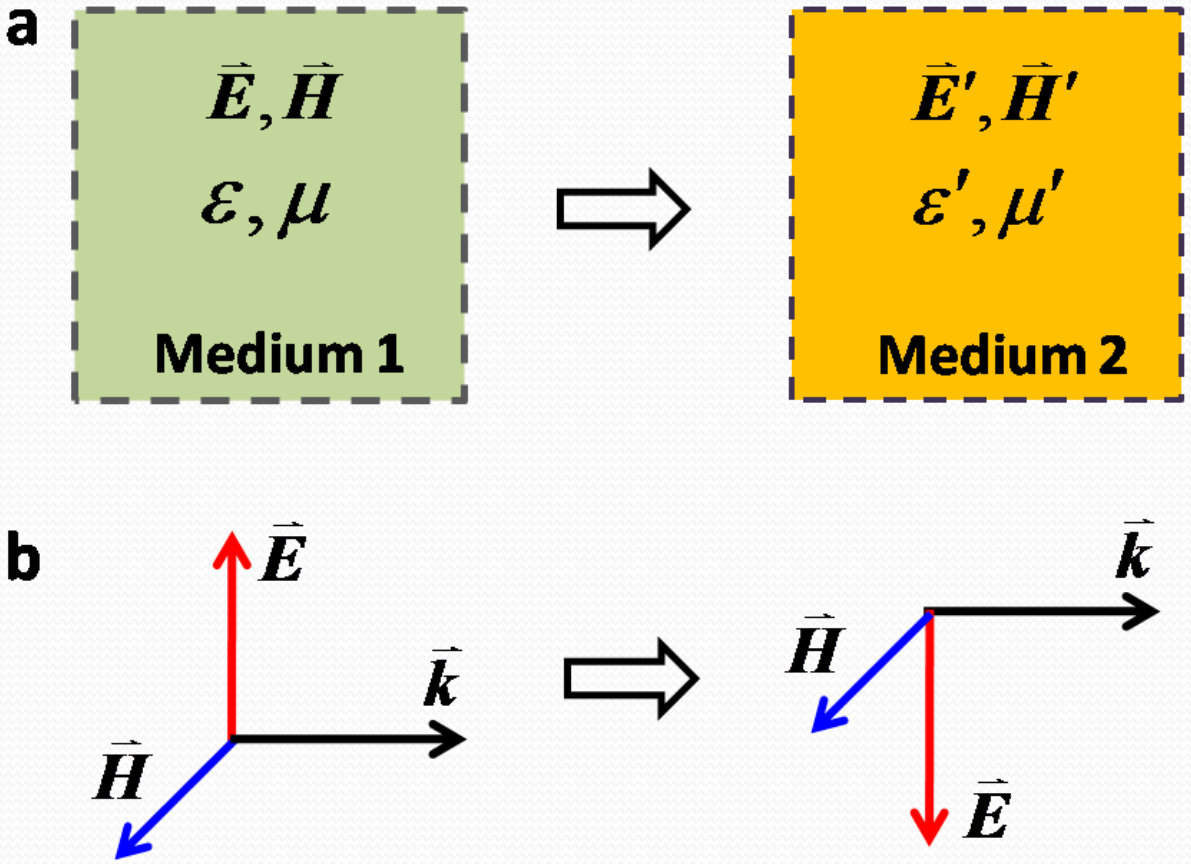
J. B. Pendry, Negative Refraction Makes a Perfect Lens, Phys. Rev. Lett. 85, 3966 (2000).

D. R. Smith and D. Schurig, Phys. Rev. Lett. 90, 077405 (2003).

$$\mu = \varepsilon = -1 + 0.001i$$



Conjugate metamaterials



$$\nabla \times \vec{E} = -i\omega\mu_0\mu\vec{H} \qquad \vec{E}' = e^{i\alpha}\vec{E}$$

$$\nabla \times \vec{H} = i\omega\varepsilon_0\varepsilon\vec{E} \qquad \vec{H}' = e^{i\beta}\vec{H}$$

$$\mu' = e^{i(\alpha-\beta)}\mu \qquad \varepsilon' = e^{-i(\alpha-\beta)}\varepsilon$$

$$n^2 = \varepsilon\mu = \varepsilon'\mu'$$

$$\varepsilon = \mu = 1 \qquad \mu' = e^{i\alpha} \qquad \varepsilon' = e^{-i\alpha}$$

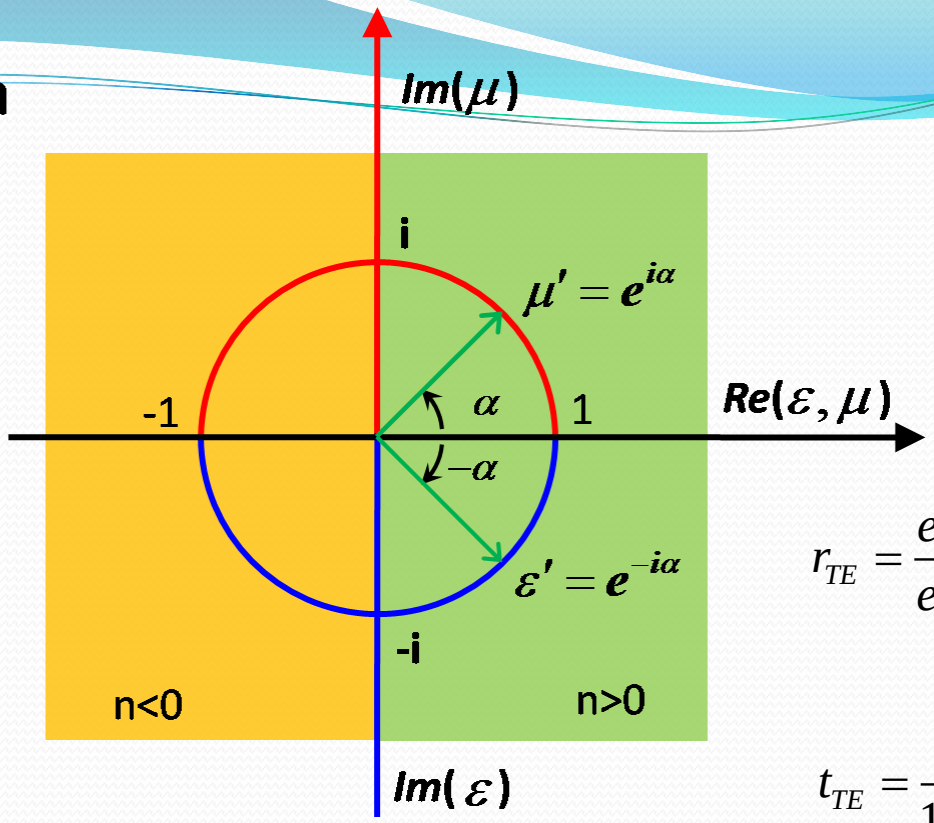
$$\alpha = \pi$$

$$\mu' = \varepsilon' = -1$$

D. Dragoman,
Optics Communications 284, 2095-2098 (2011).

Y. Xu et al, arXiv:1510.08638

a



$$r_{TE}' = \frac{e^{i\alpha}+1}{e^{i\alpha}-1} = \frac{\cot\frac{\alpha}{2}}{i}$$

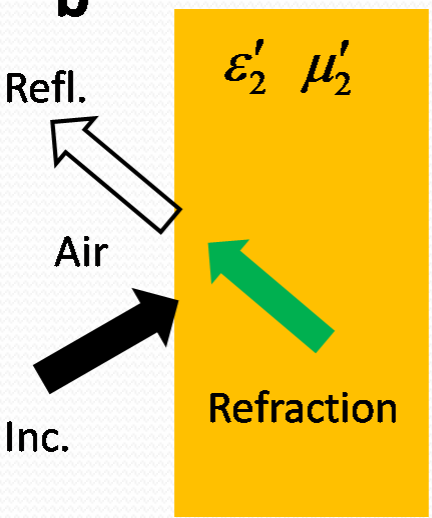
$$t_{TE}' = \frac{2e^{i\alpha}}{e^{i\alpha}-1} = \frac{\csc\frac{\alpha}{2}e^{i\frac{\alpha}{2}}}{i}$$

$$r_{TE} = \frac{e^{i\alpha}-1}{e^{i\alpha}+1} = i\tan\frac{\alpha}{2}$$

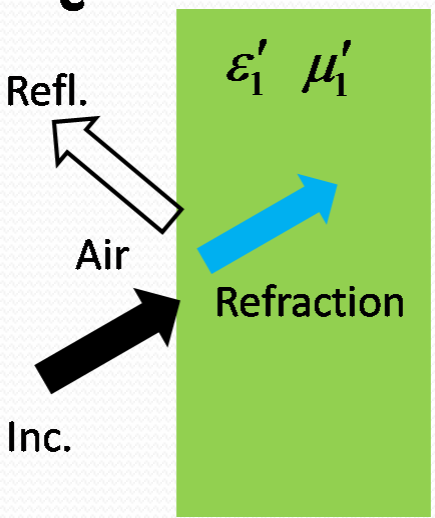
$$t_{TE} = \frac{2e^{i\alpha}}{1+e^{i\alpha}} = \sec\frac{\alpha}{2}e^{i\frac{\alpha}{2}}$$

b

$k_z' = -k_z$



c

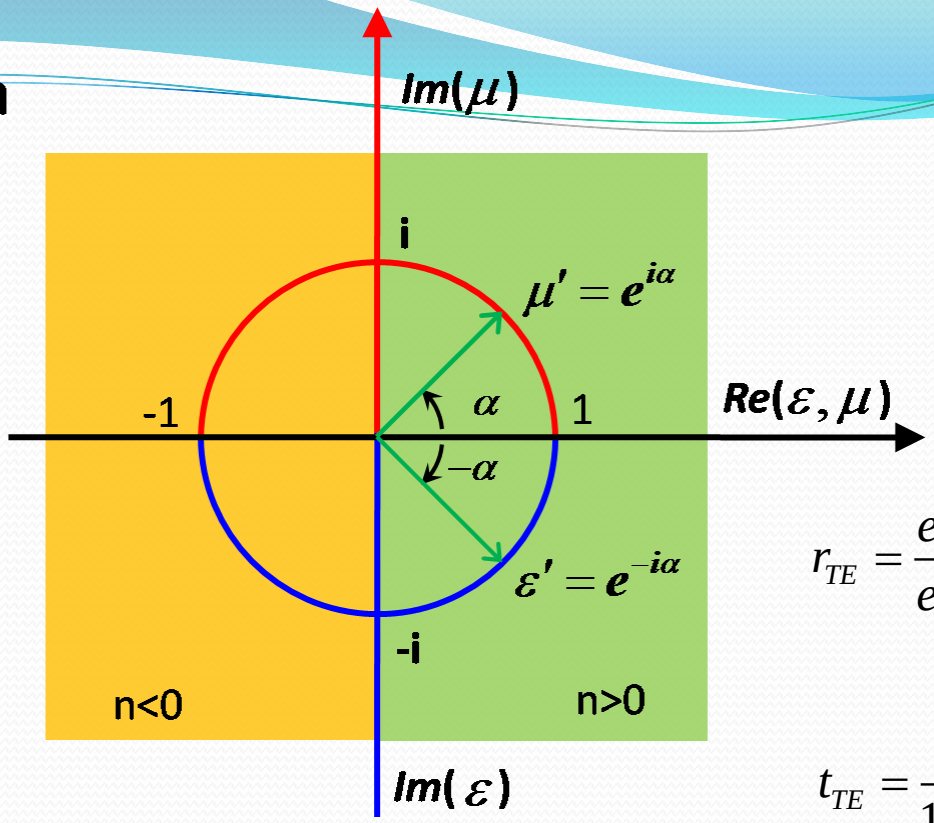


$k_z' = k_z$

$\alpha = \pi$

$r_{TE}, t_{TE} \to \infty$

a



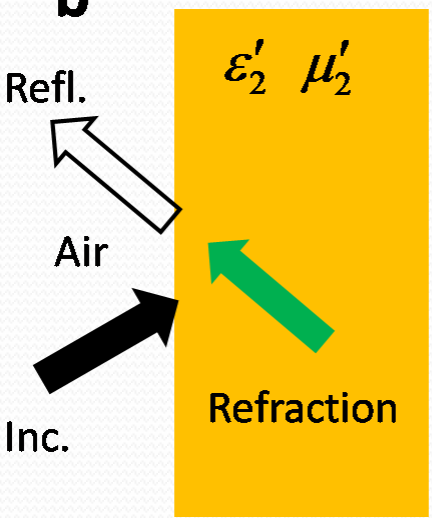
$$r_{TE}' = \frac{e^{i\alpha}+1}{e^{i\alpha}-1} = \frac{\cot\frac{\alpha}{2}}{i}$$

$$t_{TE}' = \frac{2e^{i\alpha}}{e^{i\alpha}-1} = \frac{\csc\frac{\alpha}{2}e^{i\frac{\alpha}{2}}}{i}$$

$$r_{TE} = \frac{e^{i\alpha}-1}{e^{i\alpha}+1} = i\tan\frac{\alpha}{2}$$

$$t_{TE} = \frac{2e^{i\alpha}}{1+e^{i\alpha}} = \sec\frac{\alpha}{2}e^{i\frac{\alpha}{2}}$$

b



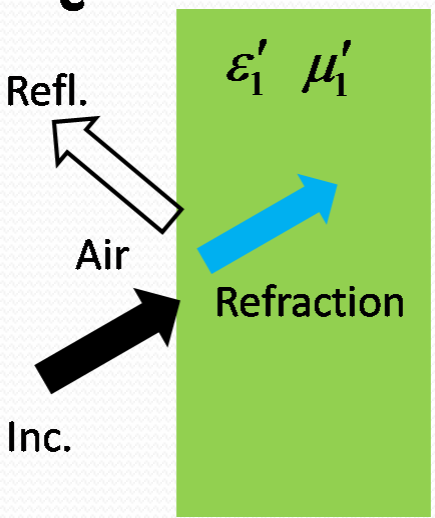
$$k_z' = -k_z$$

$$\alpha = \frac{\pi}{2}$$

$$r_{TE}' = -i$$

$$t_{TE}' = 1-i$$

c

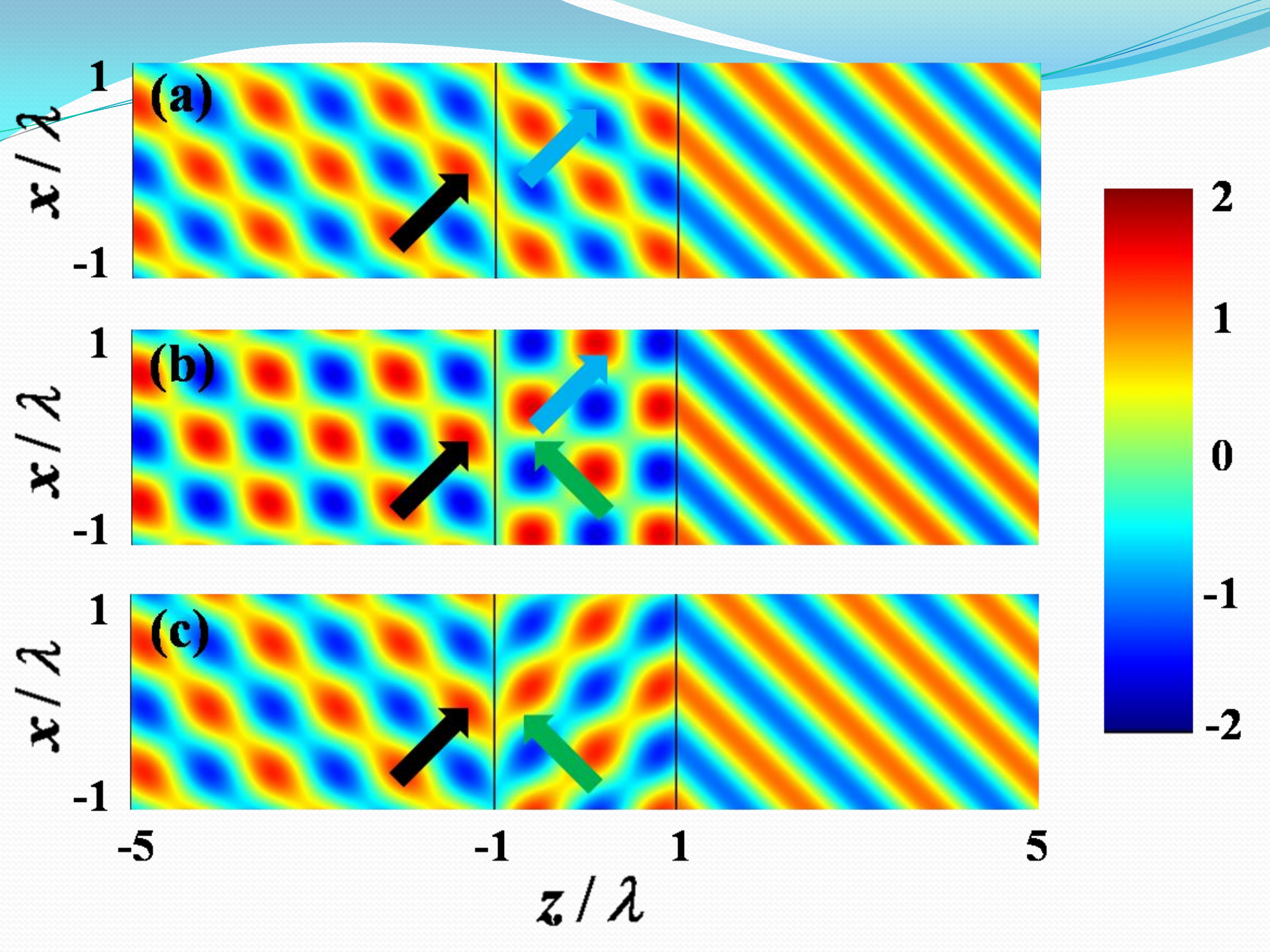


$$k_z' = k_z$$

$$\alpha = \frac{\pi}{2}$$

$$r_{TE} = i$$

$$t_{TE} = 1+i$$


(a)
(b)
(c)
1
-1
x / λ
-5
-1
1
5
z / λ
2
1
0
-1
-2

$$r_{TE} = -\frac{e^{2i\phi}-1}{\frac{1-e^{i\alpha}}{1+e^{i\alpha}}e^{2i\phi}-\frac{1+e^{i\alpha}}{1-e^{i\alpha}}} = \frac{e^{2i\phi}-1}{i\tan\frac{\alpha}{2}e^{2i\phi}+i\cot\frac{\alpha}{2}}$$

$$r_{TM} = -\frac{e^{2i\phi}-1}{\frac{1-e^{-i\alpha}}{1+e^{-i\alpha}}e^{2i\phi}-\frac{1+e^{-i\alpha}}{1-e^{-i\alpha}}} = \frac{i(e^{2i\phi}-1)}{\tan\frac{\alpha}{2}e^{2i\phi}+\cot\frac{\alpha}{2}}$$

$$t_{TE} = \frac{(1-e^{i\alpha})^2-(1+e^{i\alpha})^2}{(1-e^{i\alpha})^2e^{2i\phi}-(1+e^{i\alpha})^2}e^{i\phi} = \frac{e^{i\phi}}{\sin^2\frac{\alpha}{2}e^{2i\phi}+\cos^2\frac{\alpha}{2}}$$

$$t_{TM} = \frac{(1-e^{-i\alpha})^2-(1+e^{-i\alpha})^2}{(1-e^{-i\alpha})^2e^{2i\phi}-(1+e^{-i\alpha})^2}e^{i\phi} = \frac{e^{i\phi}}{\sin^2\frac{\alpha}{2}e^{2i\phi}+\cos^2\frac{\alpha}{2}}$$

$$a_p = \frac{2e^{i\alpha}(1+e^{i\alpha})}{(1+e^{i\alpha})^2-(1-e^{i\alpha})^2e^{2i\phi}} = \frac{\cos\frac{\alpha}{2}e^{i\frac{\alpha}{2}}}{\cos^2\frac{\alpha}{2}+e^{2i\phi}\sin^2\frac{\alpha}{2}}$$

$$a_p' = \frac{2e^{-i\alpha}(1+e^{-i\alpha})}{(1+e^{-i\alpha})^2-(1-e^{-i\alpha})^2e^{2i\phi}} = \frac{\cos\frac{\alpha}{2}e^{-i\frac{\alpha}{2}}}{\cos^2\frac{\alpha}{2}+e^{2i\phi}\sin^2\frac{\alpha}{2}}$$

$$a_n = \frac{2e^{i\alpha}(1-e^{i\alpha})e^{2i\phi}}{(1+e^{i\alpha})^2-(1-e^{i\alpha})^2e^{2i\phi}} = \frac{\sin\frac{\alpha}{2}e^{i(\frac{\alpha}{2}-\frac{\pi}{2})}e^{2i\phi}}{\cos^2\frac{\alpha}{2}+e^{2i\phi}\sin^2\frac{\alpha}{2}}$$

$$a_n' = \frac{2e^{-i\alpha}(1-e^{-i\alpha})e^{2i\phi}}{(1+e^{-i\alpha})^2-(1-e^{-i\alpha})^2e^{2i\phi}} = \frac{i\sin\frac{\alpha}{2}e^{-i\frac{\alpha}{2}}e^{2i\phi}}{\cos^2\frac{\alpha}{2}+e^{2i\phi}\sin^2\frac{\alpha}{2}}$$

$$\phi = k_z d$$

$$A_p = \frac{\cos\frac{\alpha}{2}}{|\cos^2\frac{\alpha}{2}+e^{2i\phi}\sin^2\frac{\alpha}{2}|} \qquad A_n = \frac{\sin\frac{\alpha}{2}}{|\cos^2\frac{\alpha}{2}+e^{2i\phi}\sin^2\frac{\alpha}{2}|}$$

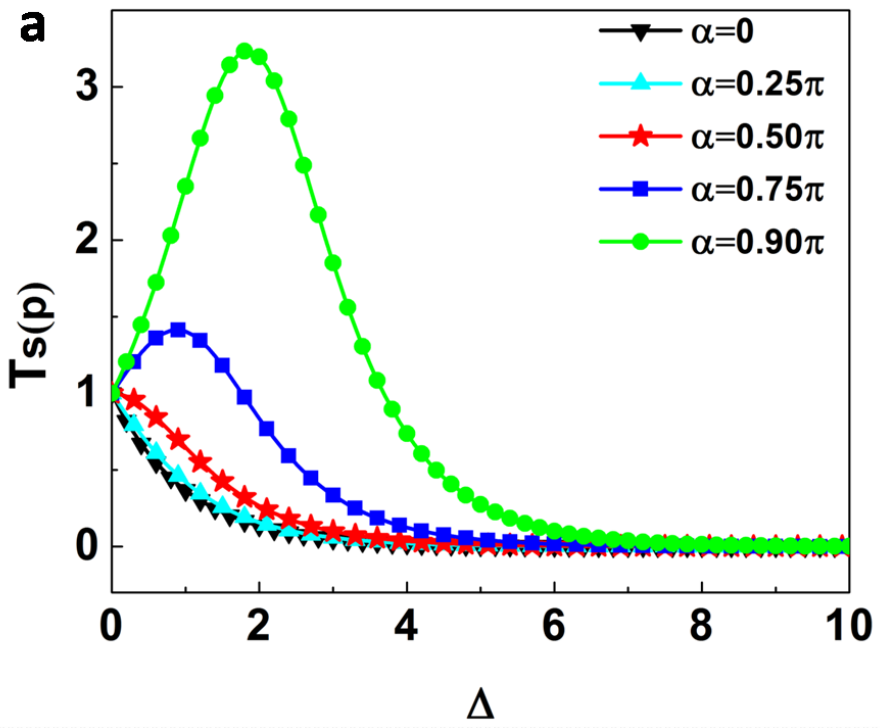
$$\alpha = 0, A_p = 1, A_n = 0$$

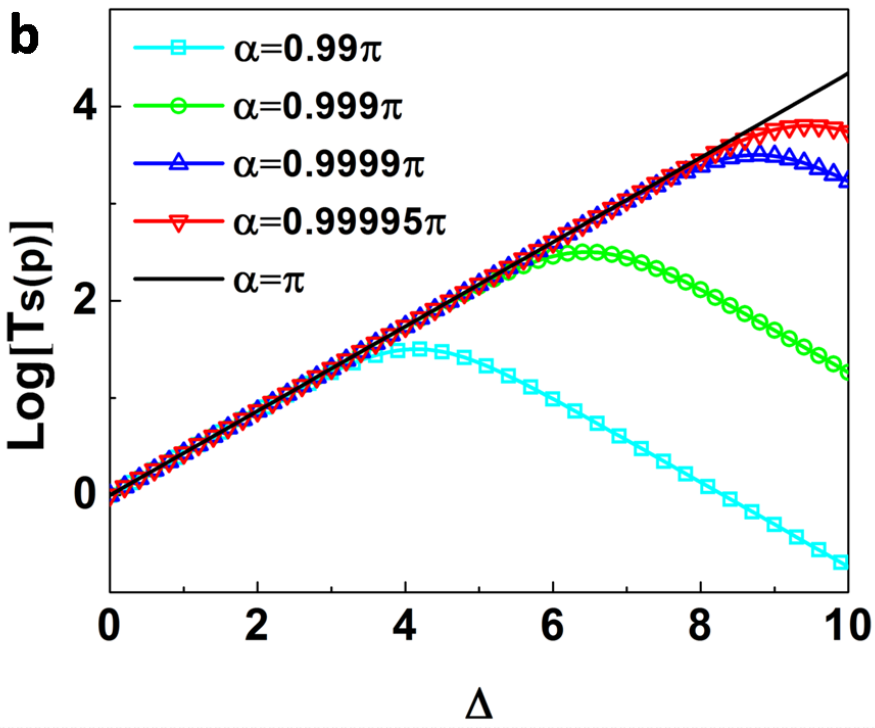
$$\alpha = \pi, A_p = 0, A_n = 1$$

$$\alpha = \frac{\pi}{2}, A_p = \frac{1}{2}, A_n = \frac{1}{2}$$

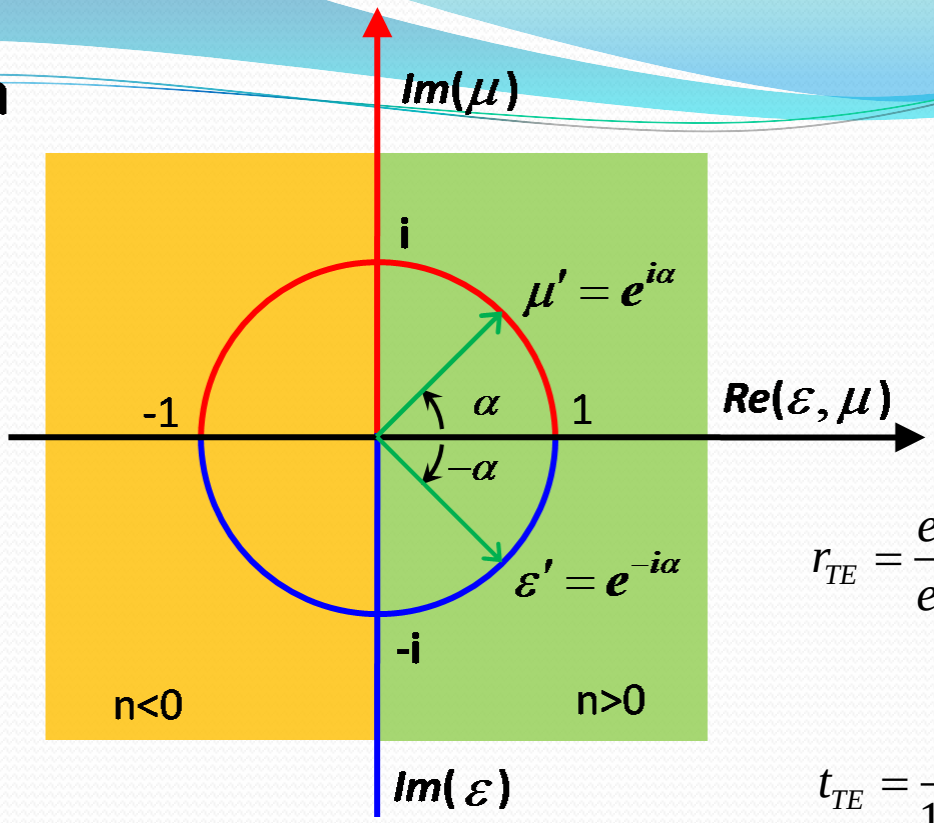
$$T_{s(p)} = \frac{\exp(ik_z d)}{\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}\exp(2ik_z d)} \qquad k_z = i\kappa$$

$$T_{s(p)} = \frac{\exp\Delta}{\exp 2\Delta \times \cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}} \qquad \Delta = \kappa d$$





a



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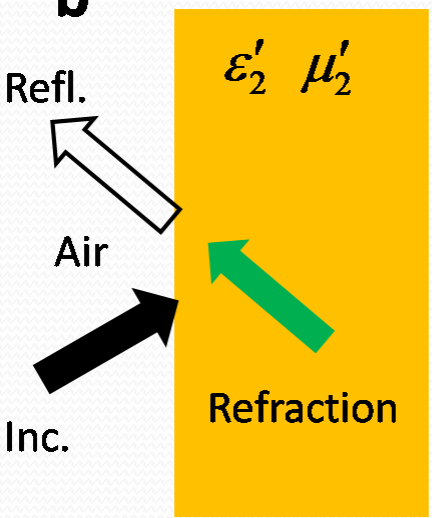
b



$$k_z' = -k_z$$

$$\alpha = \frac{\pi}{2}$$

$$r_{TE}' = -i$$

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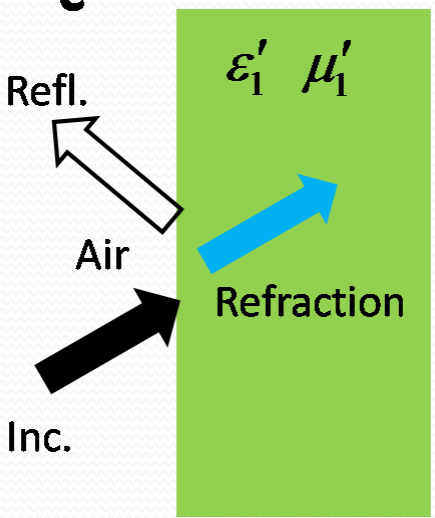
c



$$k_z' = k_z$$

$$\alpha = \frac{\pi}{2}$$

$$r_{TE} = i$$

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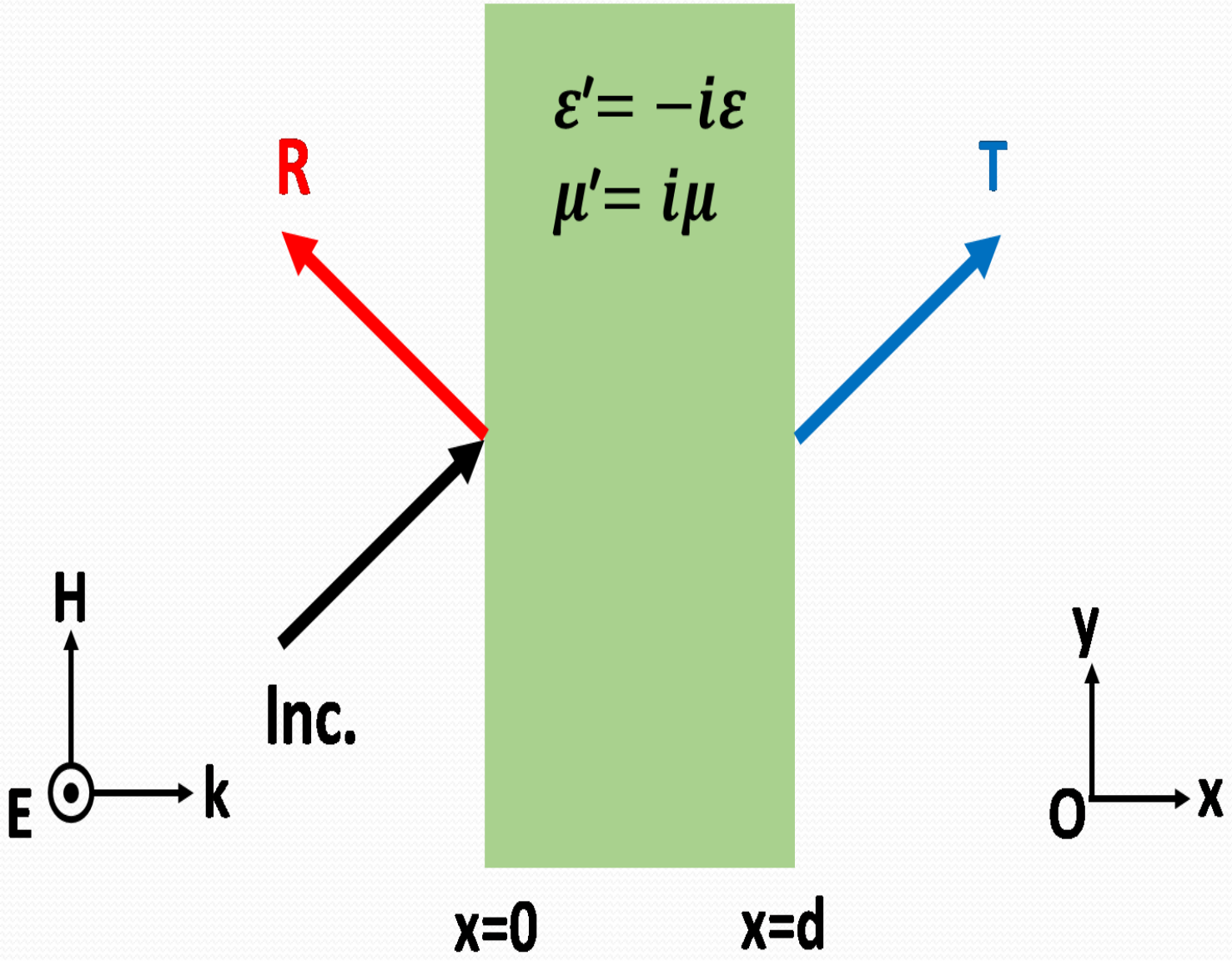

$\varepsilon' = -i\varepsilon$
$\mu' = i\mu$
R
T
H
E
k
Inc.
y
O
x
x=0
x=d

传统材料：$\varepsilon' = 0.5, \mu' = 0.5$

用TE波：

区域1：x<0，$E_1 = E_0 e^{(ik_x x + ik_y y)} + E_0 r e^{(-ik_x x + ik_y y)}$

区域2：0<x<d，$E_2 = Ae^{(ik'_x x + ik'_y y)} + Be^{(-ik'_x x + ik'_y y)}$

区域3：x>d，$E_3 = E_0 t e^{\left(ik_y y + ik_x (x-d)\right)}$

根据麦克斯韦方程和边界连续条件：

$$r = \frac{2i\left(1-\alpha^2\right)\sin\phi}{(1+\alpha)^2 e^{-i\phi} - (1-\alpha)^2 e^{i\phi}}$$

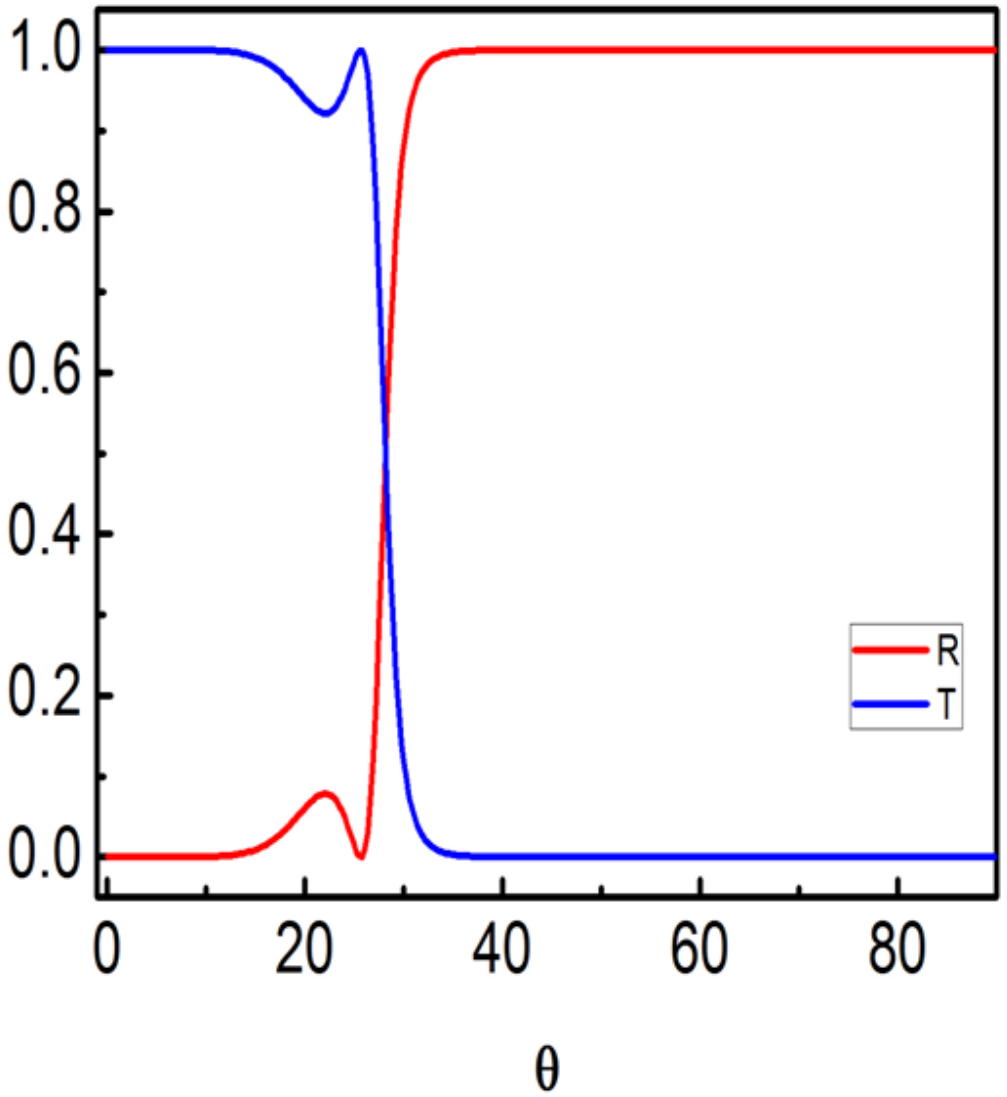
$$t = \frac{4\alpha}{(1+\alpha)^2 e^{-i\phi} - (1-\alpha)^2 e^{i\phi}}$$

$$\theta \le \theta_c, \theta_c = arcsin0.5 = 30^\circ, \alpha = \frac{\mu\cos\theta}{\sqrt{0.25 - \sin^2\theta}}, \quad \phi = k_0 d\sqrt{0.25 - \sin^2\theta}$$

$$\theta > \theta_c, \ \alpha = \frac{-i\mu\cos\theta}{\sqrt{\sin^2\theta - 0.25}}, \ \phi = ik_0 d\sqrt{\sin^2\theta - 0.25}$$

取 $d = 2$

$\lambda = 1$



共轭材料： $\varepsilon' = -i\varepsilon \quad \mu' = i\mu$

区域1：x<0, $E_1 = E_0 e^{(ik_x x + ik_y y)} + E_0 r e^{(-ik_x x + ik_y y)}$

区域2：0<x<d, $E_2 = Ae^{(ik'_x x + ik'_y y)} + Be^{(-ik'_x x + ik'_y y)}$

区域3：x>d, $E_3 = E_0 t e^{\left(ik_y y + ik_x (x-d)\right)}$

根据麦克斯韦方程和边界连续条件：

$\theta \le \theta_c$, $\theta_c = \arcsin\sqrt{\varepsilon\mu}$

$$r_1 = \frac{(1+\beta^2)\sin\emptyset}{2\beta\cos\emptyset + (\beta^2-1)\sin\emptyset}$$

$$t_1 = \frac{2\beta}{2\beta\cos\emptyset + (\beta^2-1)\sin\emptyset}$$

其中 $\beta = \dfrac{\mu\cos\theta}{\sqrt{\varepsilon\mu - \sin^2\theta}}$, $\phi = k_0 d\sqrt{\varepsilon\mu - \sin^2\theta}$

$\theta > \theta_c$

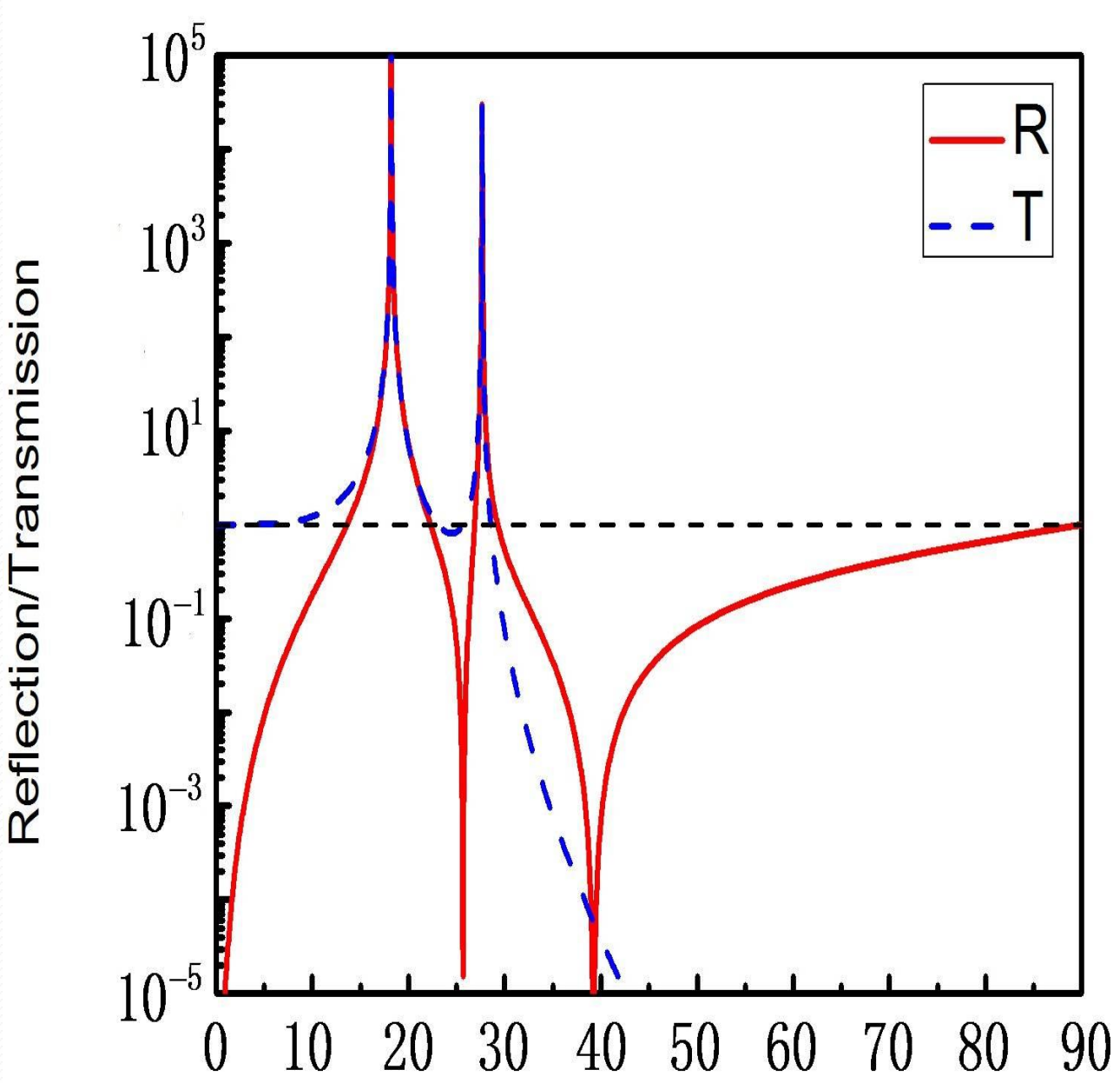
$$r_2 = \frac{(1-\alpha^2)(1-e^{2\gamma})}{(1+\alpha)^2 e^{2\gamma} - (1-\alpha)^2}$$

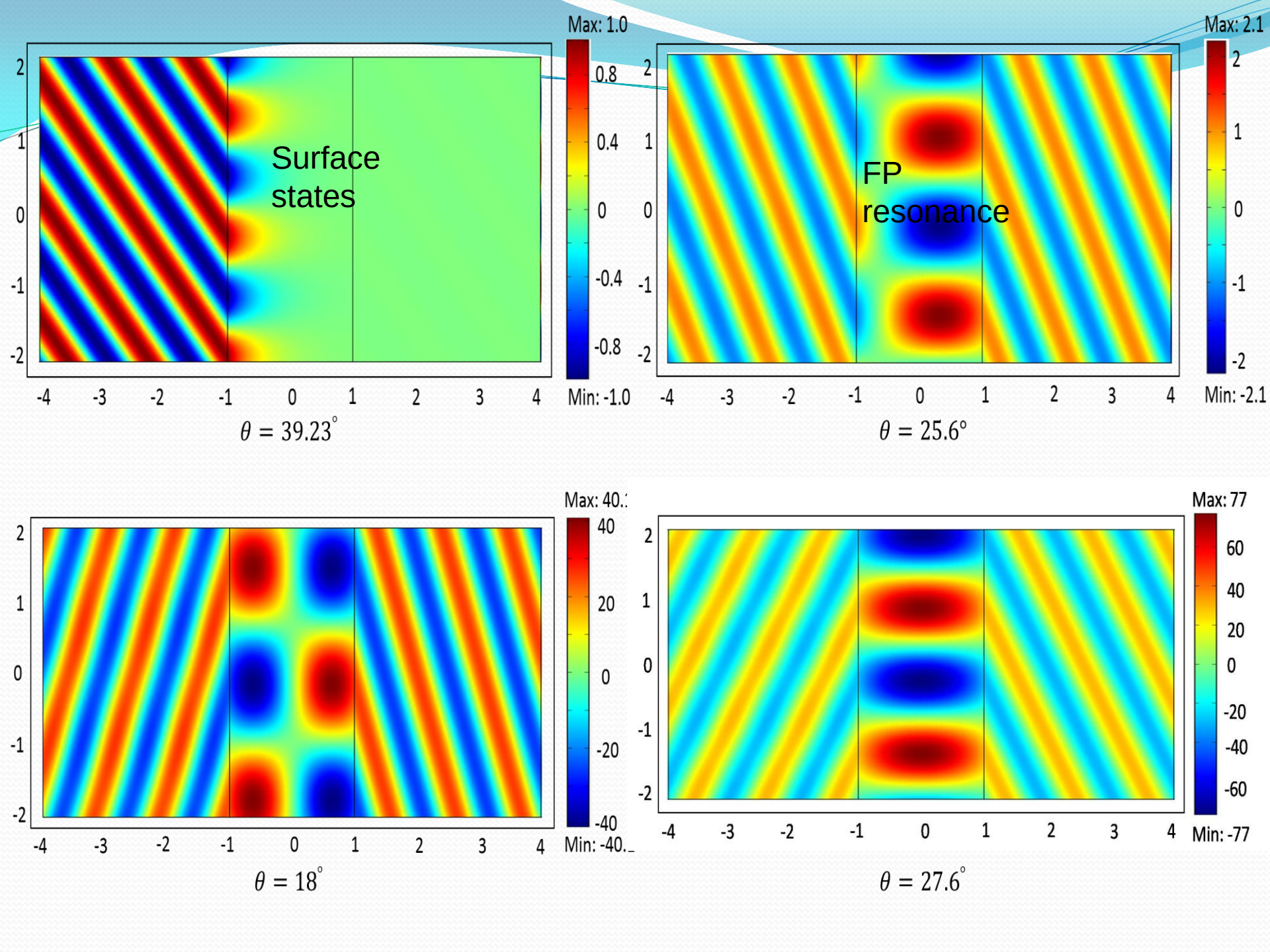
$$t_2 = \frac{4\alpha e^{\gamma}}{(1+\alpha)^2 e^{2\gamma} - (1-\alpha)^2}$$

其中 $\alpha = \dfrac{\mu\cos\theta}{\sqrt{\sin^2\theta - \varepsilon\mu}}$ $\gamma = k_0 d\sqrt{\sin^2\theta - \varepsilon\mu}$

取 $d = 2$
$\lambda = 1$
$\mu = \varepsilon = 0.5$

则 $k_0 = 2\pi$
$\theta_c = 30°$



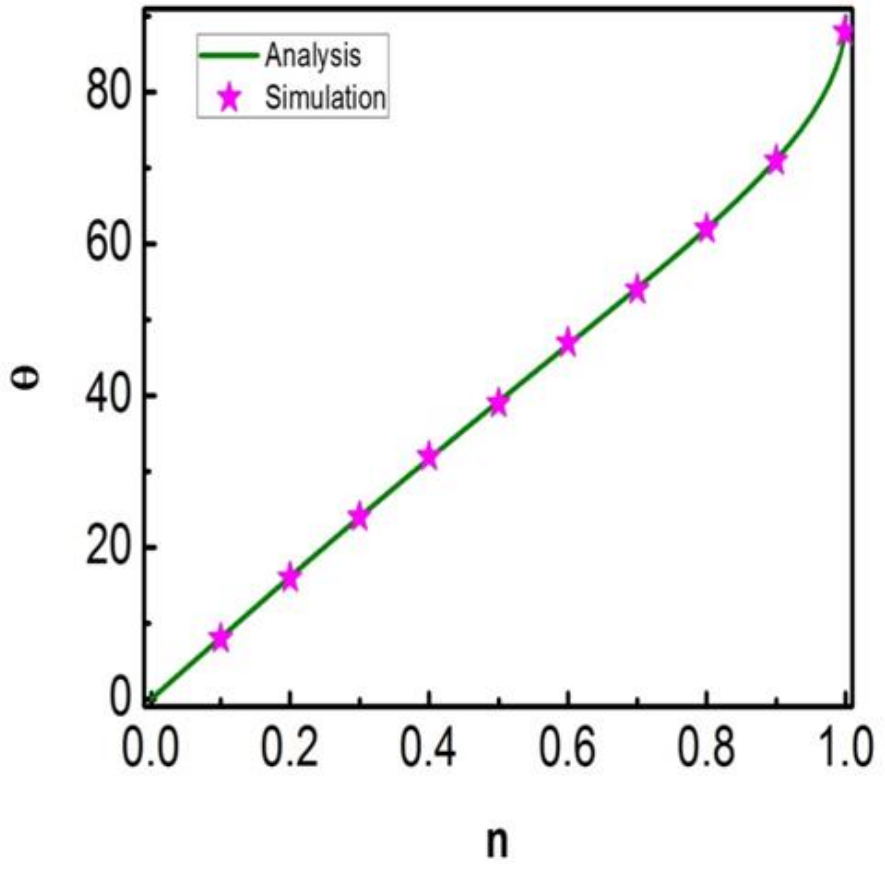

Surface states
FP resonance
Max: 1.0
Min: -1.0
Max: 2.1
Min: -2.1
Max: 40.
Min: -40.
Max: 77
Min: -77
θ = 39.23°
θ = 25.6°
θ = 18°
θ = 27.6°

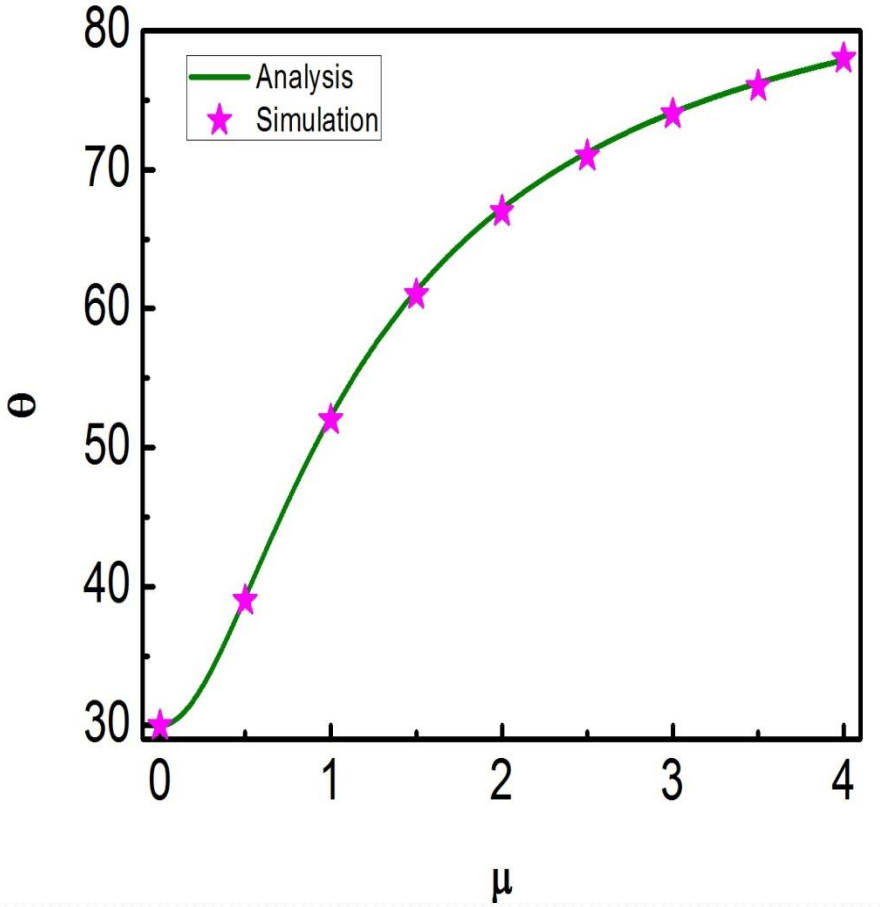
现在，取$\mu' = \varepsilon'^{*} = ni$，$0 < n < 1$

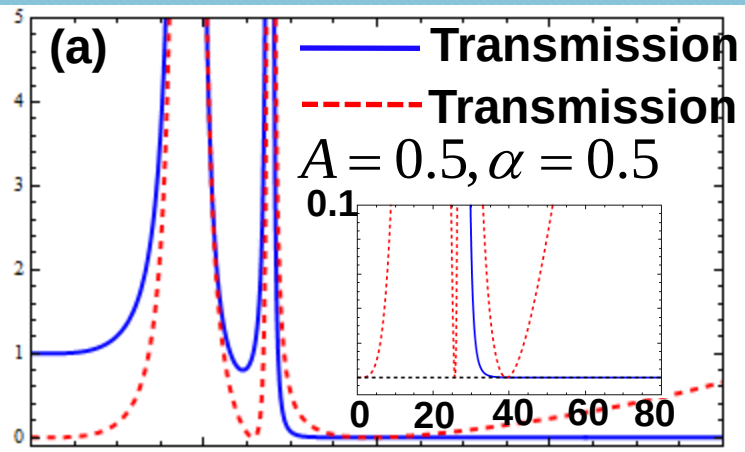
令$r = 0$，得：$\theta = arcsin\sqrt{\frac{2n^2}{n^2+1}}$

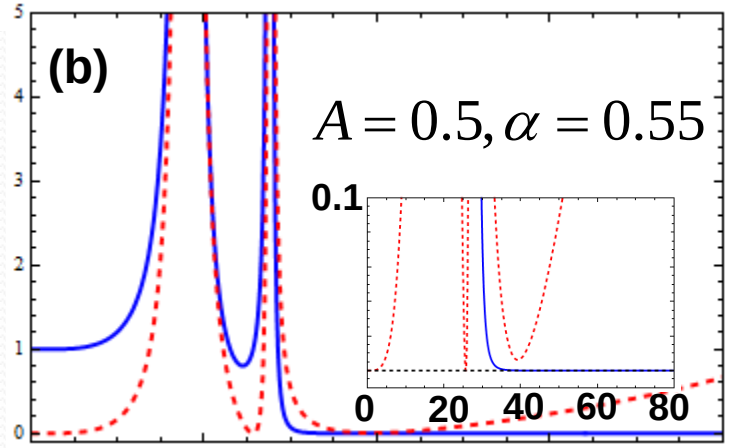
取$\varepsilon\mu = 0.25$，令$r = 0$，得：$\theta = arcsin\sqrt{\frac{\mu^2+0.25}{\mu^2+1}}$

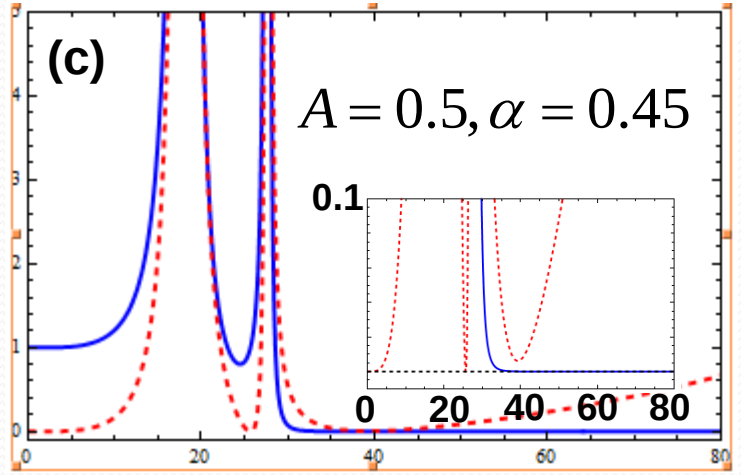
$0 < \mu \leq 4$（$\mu \to +\infty$，$\theta \to 90^{\circ}$，变化缓慢）







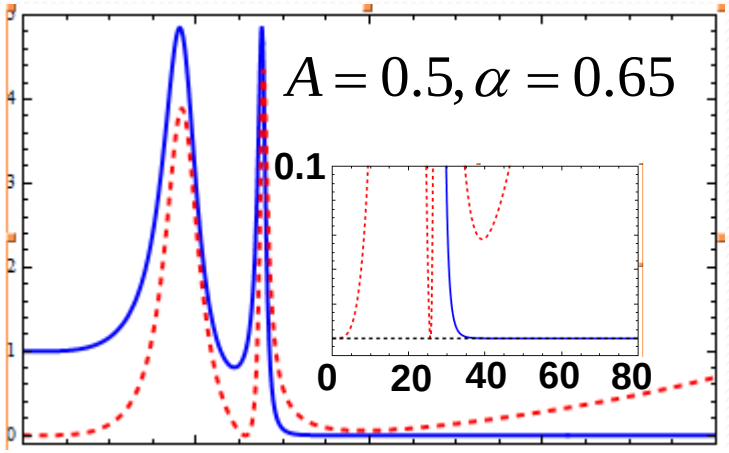


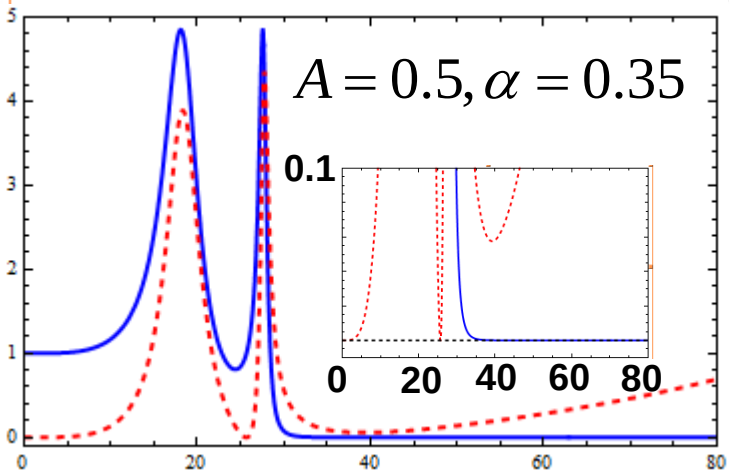


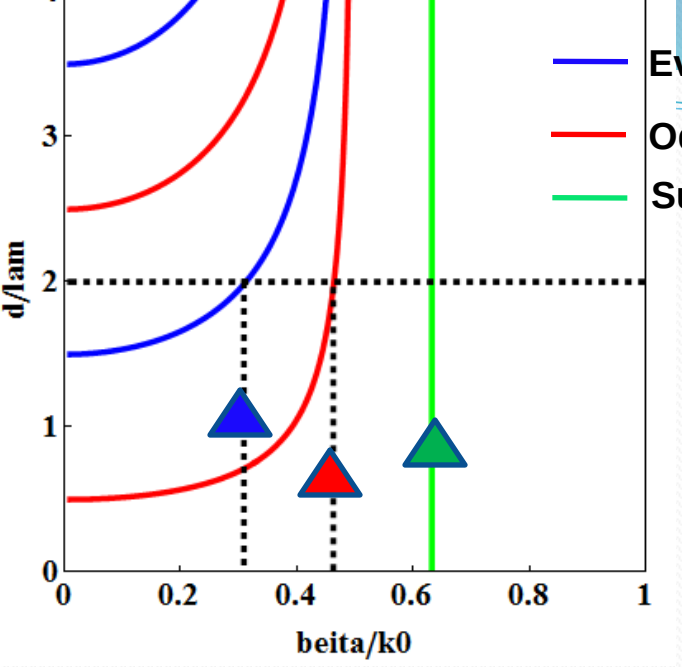
Yanyan Cao, Yangyang Fu, Yadong Xu, and Huanyang Chen, unpublished.

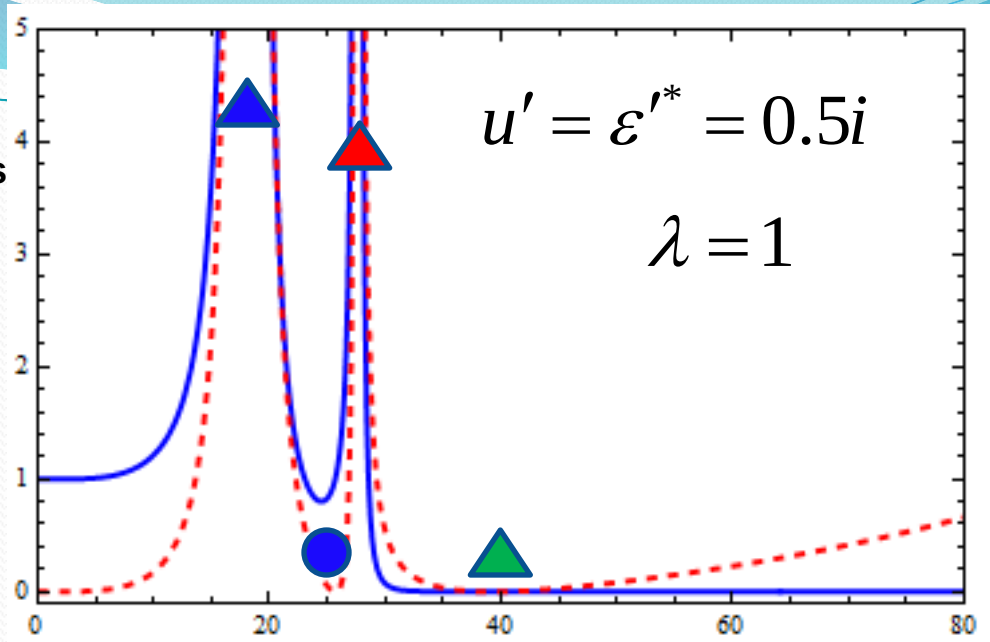
$$\varepsilon = A\exp(\mathrm{i}\,\alpha\pi)$$

$$\mu = A\exp(-\mathrm{i}\,\alpha\pi)$$

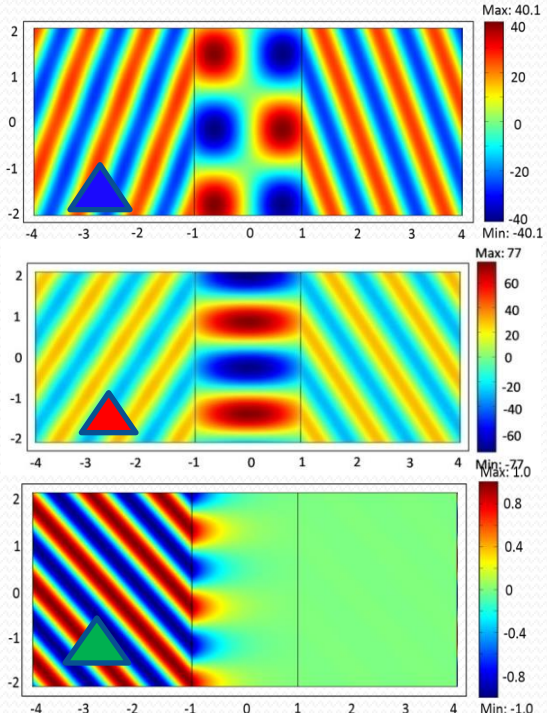


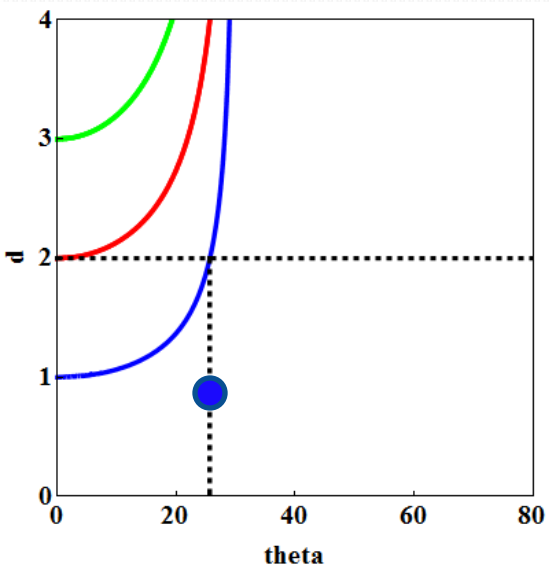


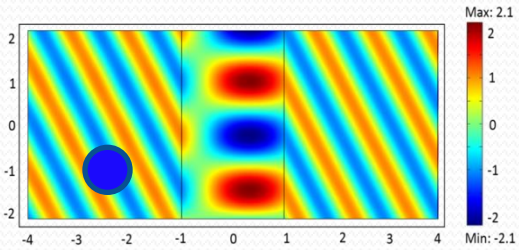




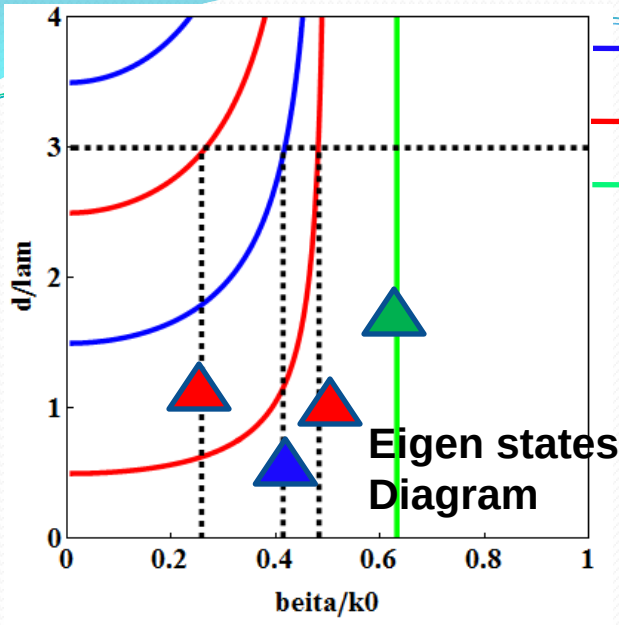
Eigen states Diagram

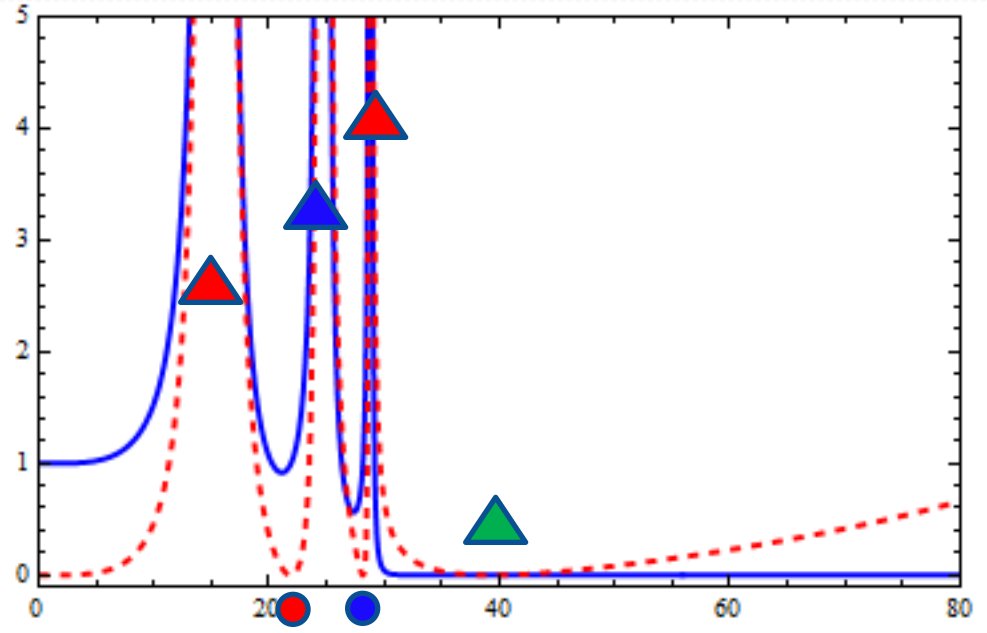


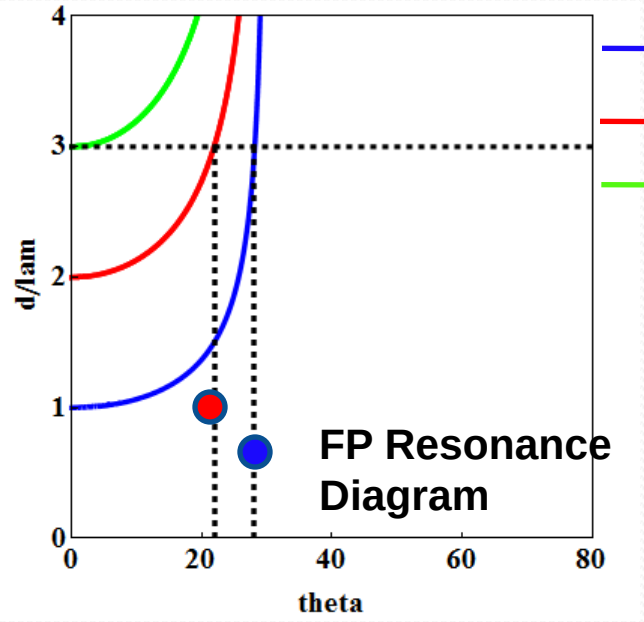


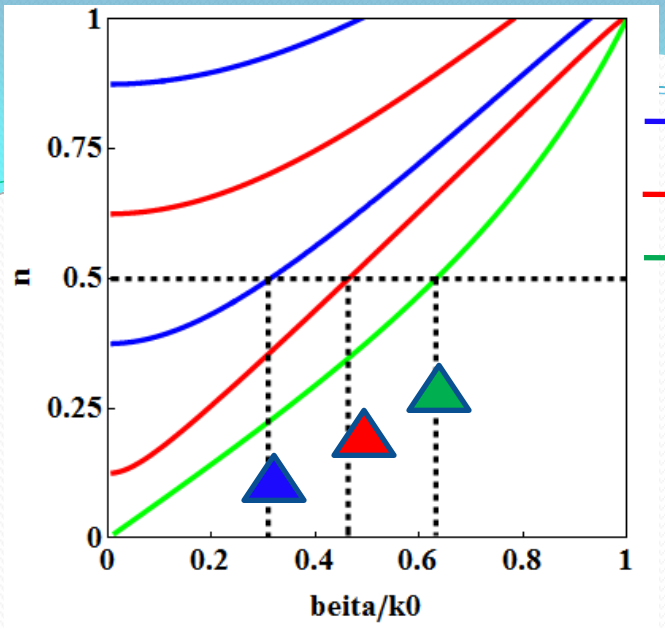


FP Resonance Diagram


Even modes
Odd modes
Surface modes
d/lam
beita/k0
Eigen states
Diagram




m=1
m=2
m=3
d/lam
theta
FP Resonance
Diagram



Even modes

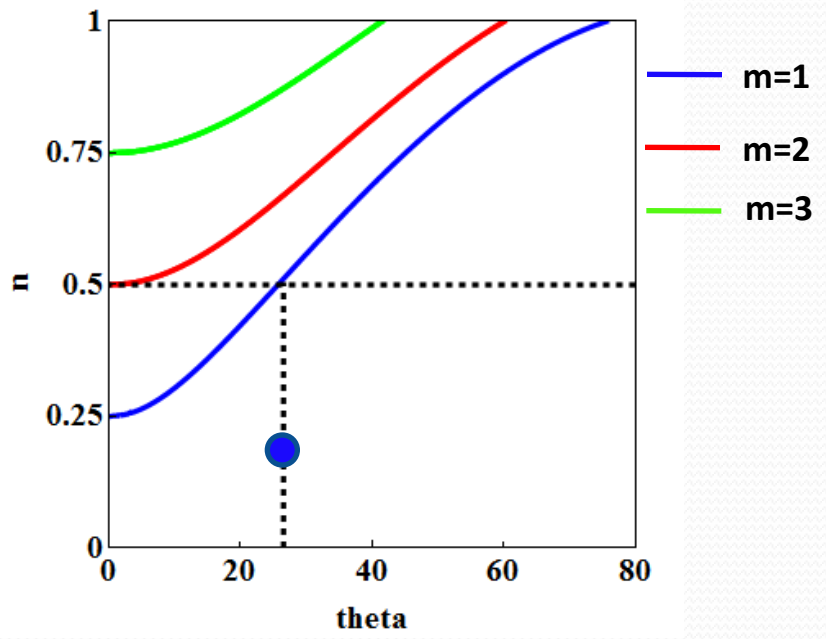
Odd modes

Surface modes

$$\lambda = 1, d = 2$$

$$u' = \varepsilon'^{*} = ni, 0 < n < 1$$

Eigen states Diagram

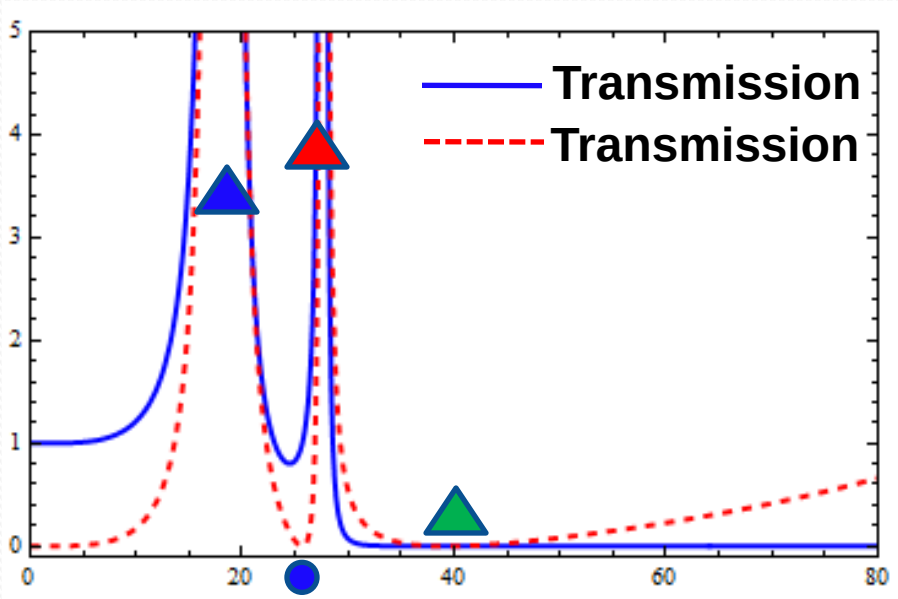


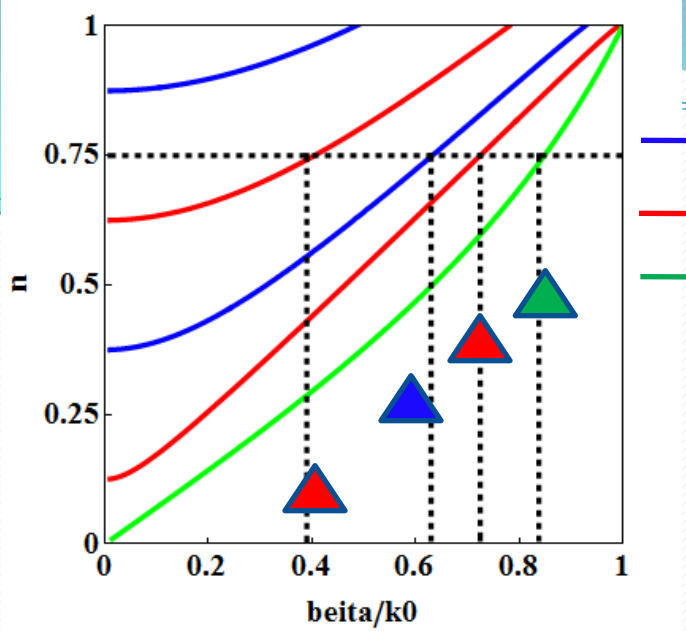
m=1

m=2

m=3

FP Resonance Diagram



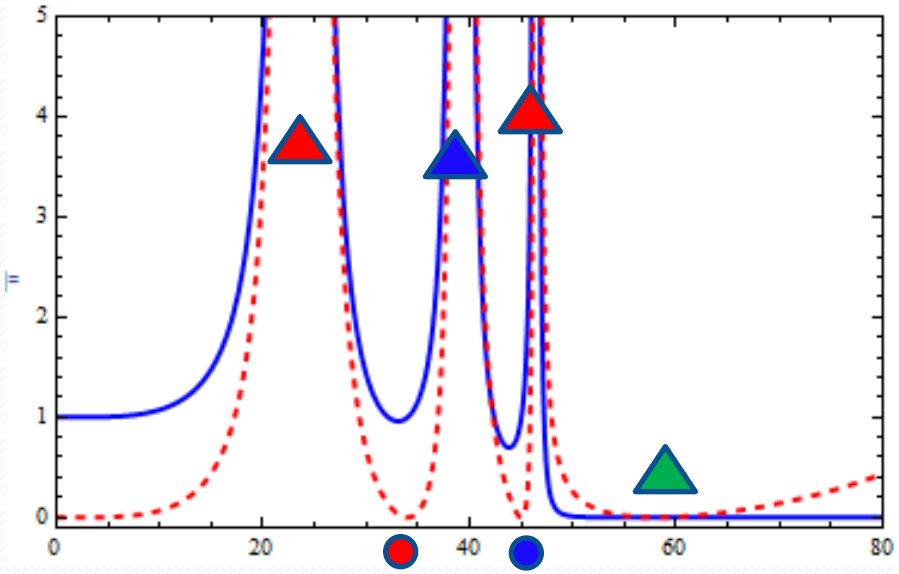


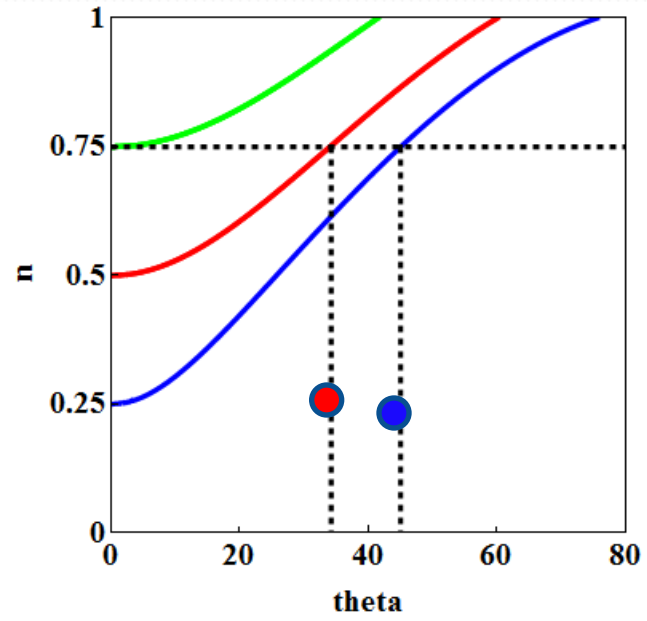
Even modes

Odd modes

Surface modes

Eigen states Diagram





m=1

m=2

m=3

FP Resonance Diagram

Conclusions

- Transformation optics makes it possible to design conjugate metamaterials.
- Perfect lens is the limiting case, tiny deviations will produce imperfections in the imaging functionality.
- Purely imaginary metamaterials support perfect conversions from propagating waves to evanescent waves, lasing modes, and FP resonances, if the incident momentum is matched.

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