Unidirectional Phenomena in Optic/Acoustic PT Materials

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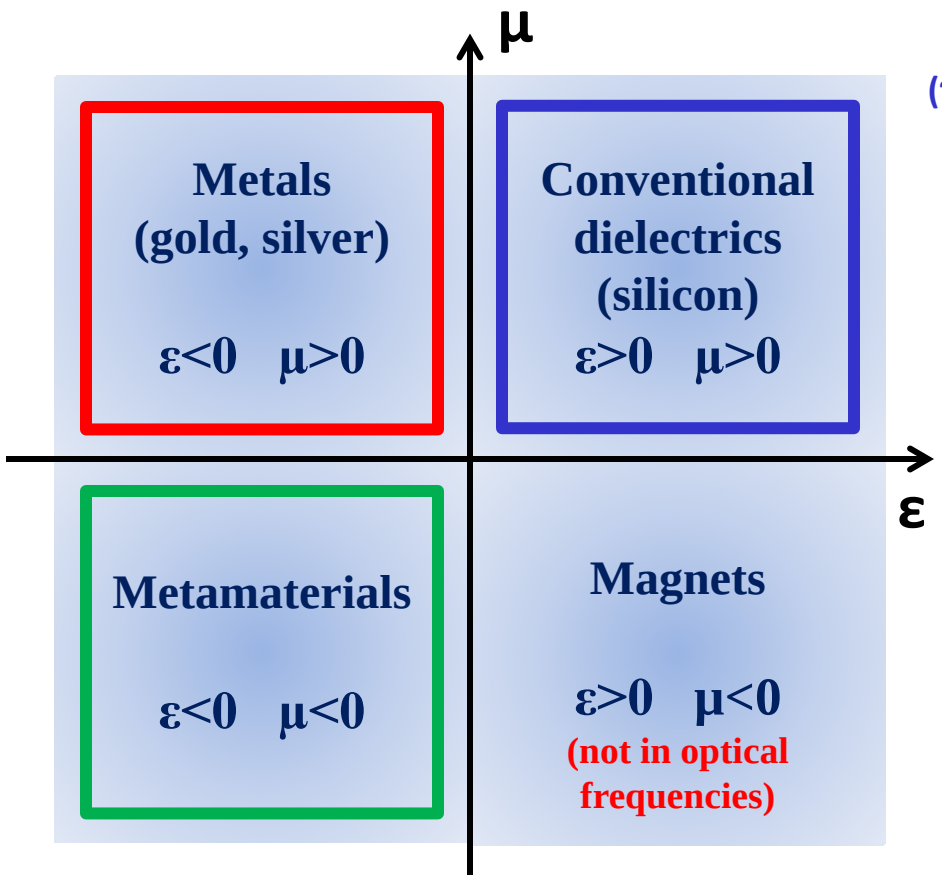
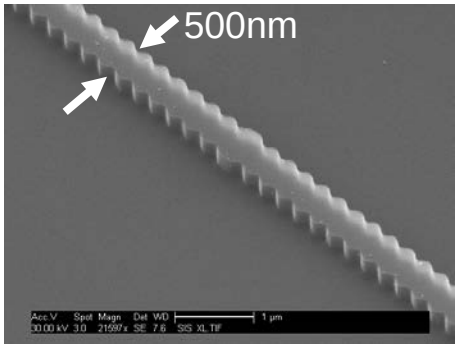
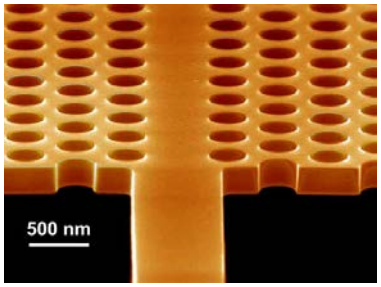
Engineering Photonics using Lithography

- Engineering photonics in a real ε-μ plane



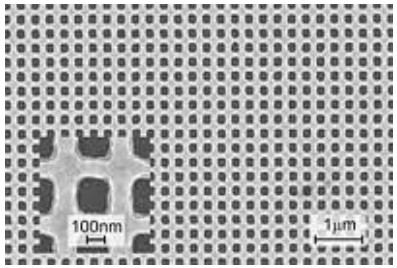
Photonic waveguides
~ λ
(~ 500 nm for Si at 1550 nm)

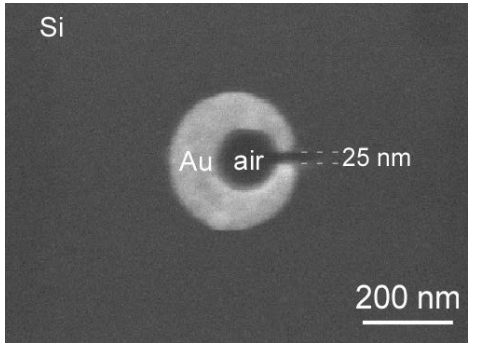




Photonic crystals ~ λ/2
(~ 300-500 nm for Si at 1550 nm)

Plasmonics ~ λ/10
(<50 nm for visible light)



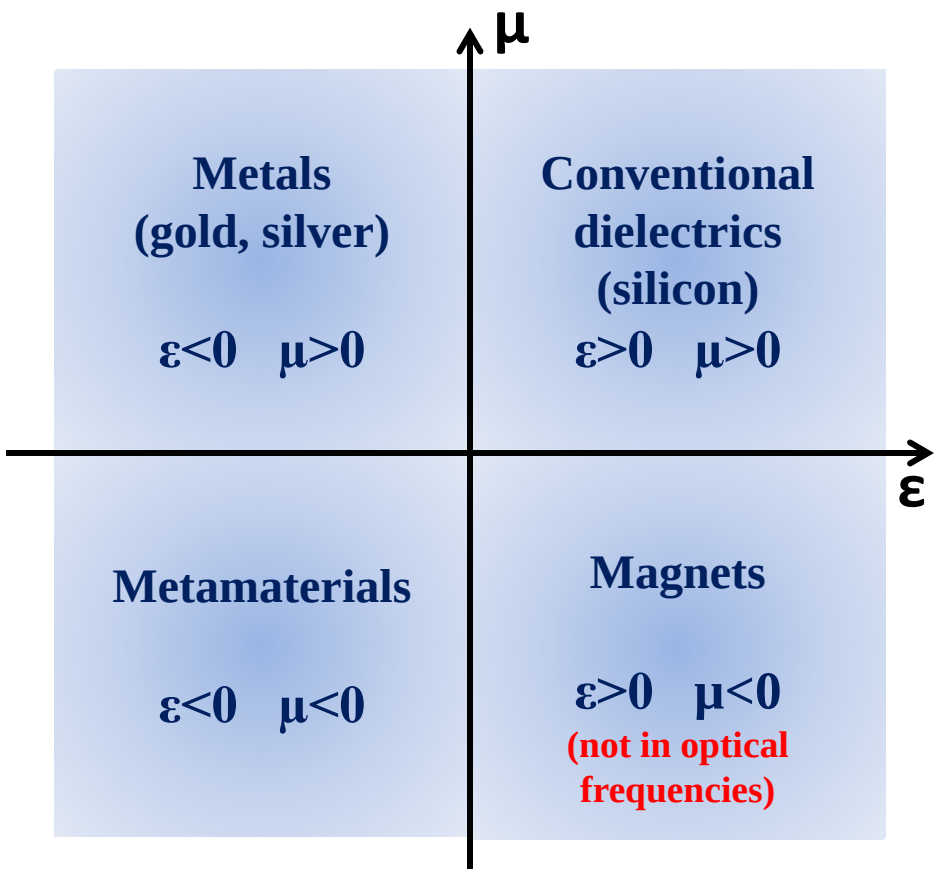


Metamaterials ~ λ/5
(<100 nm for visible light)

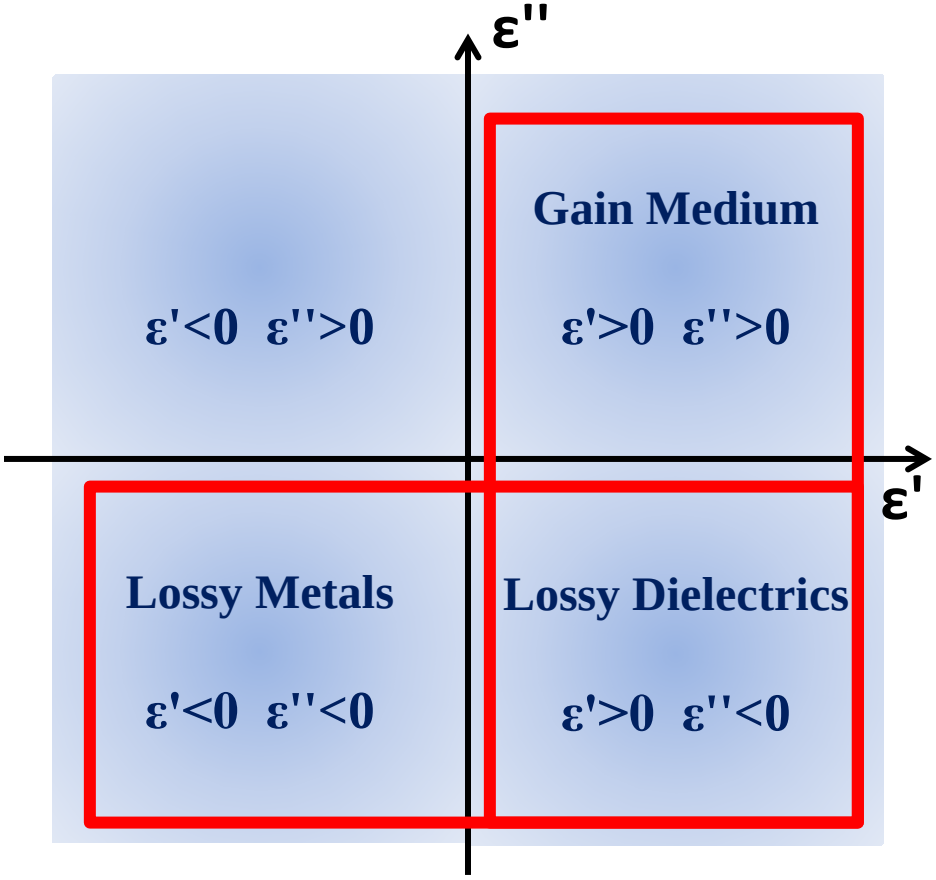
Demand nanofabrication to engineer photonics!

Photonics is More Than Real

- Photonics in a real ε-μ plane



- What is missing?



Engineering nanophotonics in a complex dielectric permittivity plane!

Parity-time (PT) Symmetry in Optics

Electron

Time-dependent Schrodinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)+\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t)-V(\mathbf{r},t)\Psi(\mathbf{r},t)=0$$

Hamiltonian in quantum mechanics

$$\hat{H}=\hat{p}^2/2m+V_R(\hat{x})+iV_I(\hat{x})$$

Parity-time symmetry in quantum mechanics

$$PT \text{ symmetry} \rightarrow PT\hat{H}=\hat{H}PT$$

Parity-time potentials in quantum mechanics

$$PT \text{ symmetry} \rightarrow V_R(\hat{x})=V_R(-\hat{x})$$
$$V_I(\hat{x})=-V_I(-\hat{x})$$

Photon

Paraxial equation of diffraction

$$ik_0^{-1}\frac{\partial}{\partial z}E+\frac{k_0^{-2}}{2(k/k_0)}\frac{\partial^2 E}{\partial x^2}+[n_R(x)+in_I(x)]E=0$$

Parity-time optical potentials

$$PT \text{ symmetry} \rightarrow n_R(x)=n_R(-x)$$
$$n_I(x)=-n_I(-x)$$

e.g. n=const.+exp($i\beta x$)

Parity and Time Operators

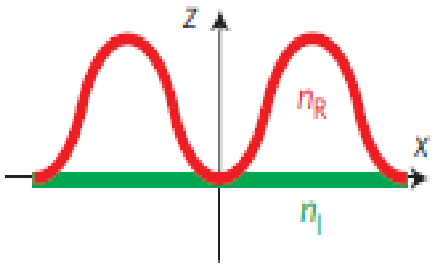
$$P:\hat{p}\rightarrow-\hat{p},\hat{x}\rightarrow-\hat{x}$$
$$T:\hat{p}\rightarrow-\hat{p},\hat{x}\rightarrow\hat{x},i\rightarrow-i$$

C. E. Ruter et al., Nature Phys. 6, 192 (2010)

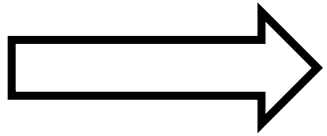
Non-Hermitian Optics

Engineering material index from real to complex: PT symmetry in optics

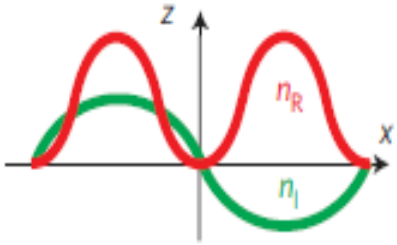
Hermitian



Index modulation
from real to complex

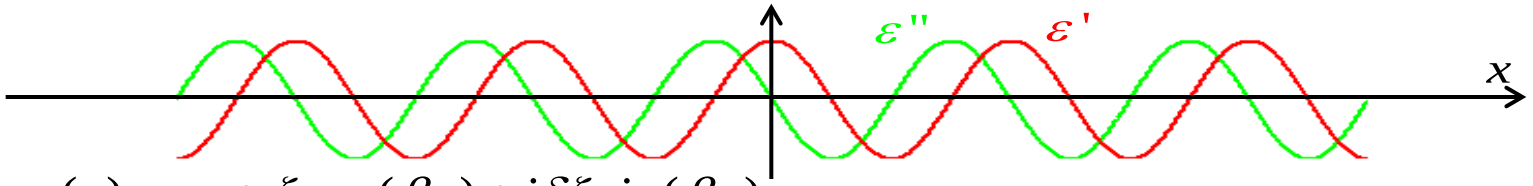


non-Hermitian



$$\varepsilon = n^2$$

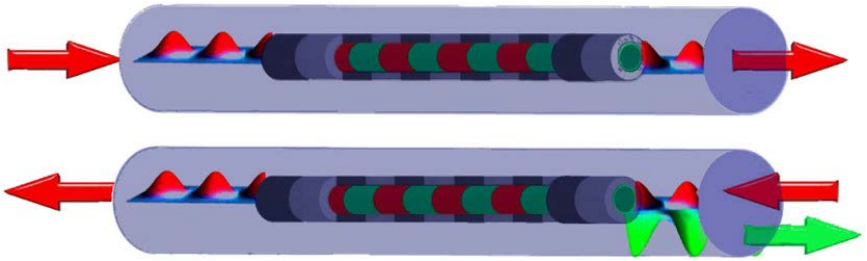
PT-optical modulation along light propagation



$$\varepsilon(x) = \varepsilon_0 + \xi\cos(\beta x) + i\delta\xi\sin(\beta x)$$

Unidirectional invisibility (δ=1)

$$\delta = 1: T = 1,\ R_L = 0$$



Z. Lin et al., Phys. Rev. Lett. 106, 213901 (2011)

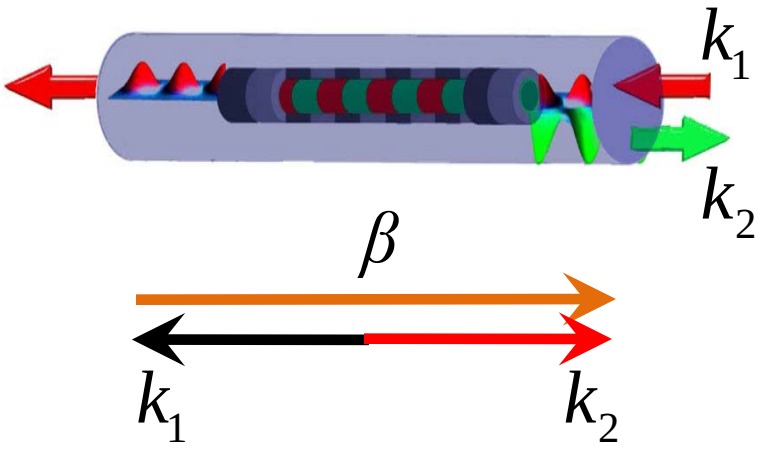
Unidirectional Reflection at δ=1

- The PT optical modulation becomes

$$\Delta\,\varepsilon = \xi \exp(i\beta x)$$

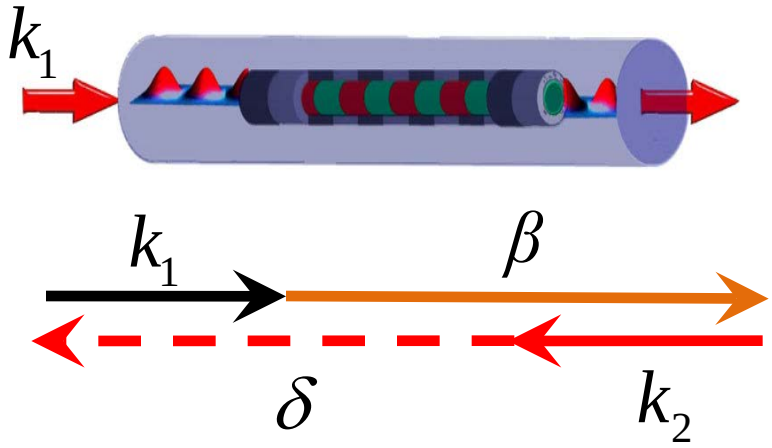
Fourier space: Introducing a unidirectional wave vector of β

- Unidirectional Bragg reflection



Phase match: $\delta=\beta+k_1-k_2=0$

(Reflection)



Phase mismatch: $\delta=\beta+k_1-k_2 \neq 0$

(Reflectionless)

Design of One-way Invisible Cloak

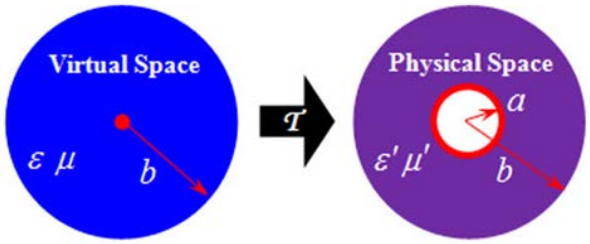
To Combine Transformation Optics with Non-Hermitian Optics:

Coordinate mapping in TO

$r = f(r') = b(r'-a)/(b-a)$

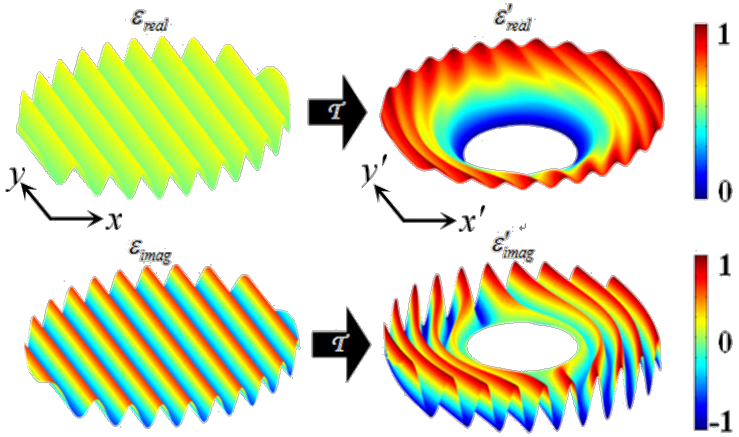
$\theta = \theta'$

ε=1, μ=1



Add PT-optical modulation into virtual space

ε=1+ξexp($i\beta x$), μ=1



$\varepsilon' = A\varepsilon A^T/detA$

$\mu' = A\mu A^T/detA$

$A_{ij} = h_i'\partial x_i'/h_i\partial x_i$

For TE light

$\varepsilon' = f(r')f'(r')\ \varepsilon/r'$

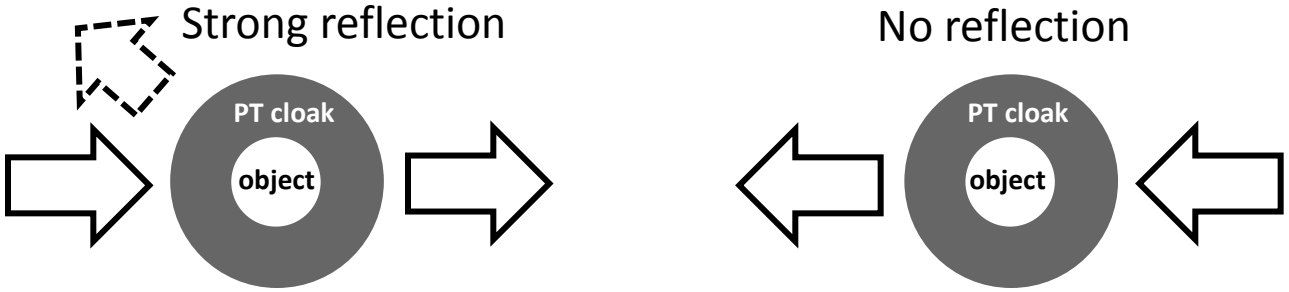
$\mu'_{r'} = f(r')\mu/[r'f'(r')]$

$\mu'_{\theta'} = f'(r')r'\mu/f(r')$

X. Zhu et al., Opt. Lett., 38, 15, 2821(2013)

X. Zhu et al., Phys. Rev. Lett. 106, 014301 (2011)

One-way Cloak for Plane Waves



For plane waves from the left

Phase match: $\delta=\beta+k_1-k_2=0$

$$\frac{dE_{in}}{dx}=0,\ \frac{dE_{sc}}{dx}=i\frac{\omega^2\xi}{2k_2c^2}E_{in}$$

(reflecting)

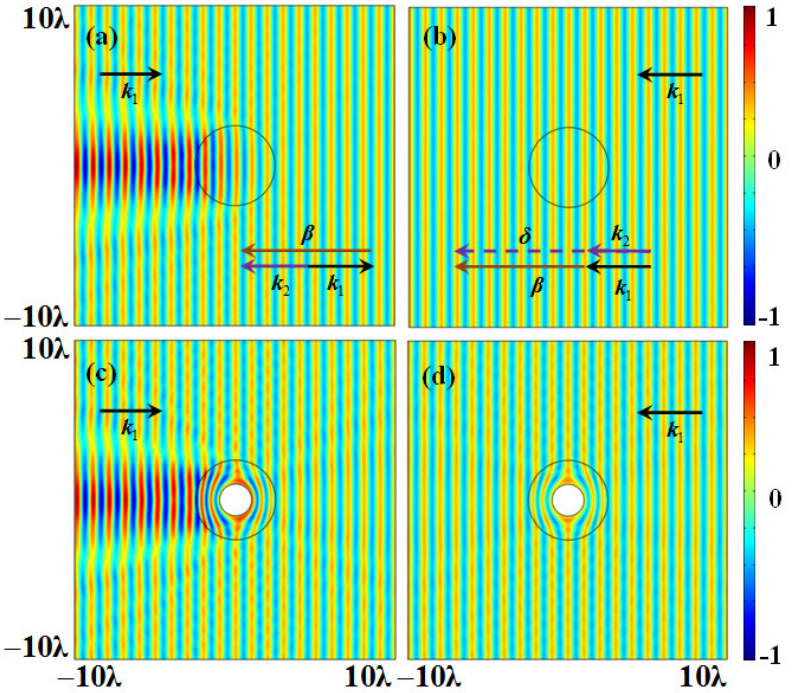
For plane waves from the right

Phase mismatch: $\delta=\beta+k_1-k_2\neq 0$

$$\frac{dE_{in}}{dx}=0,\ \frac{dE_{sc}}{dx}=0$$

(no reflecting)

Unidirectional light reflection (backward)



One-way Cloak with Forward Reflection

For plane waves from the left

Phase match: $\delta=\beta+k_1-k_2=0$

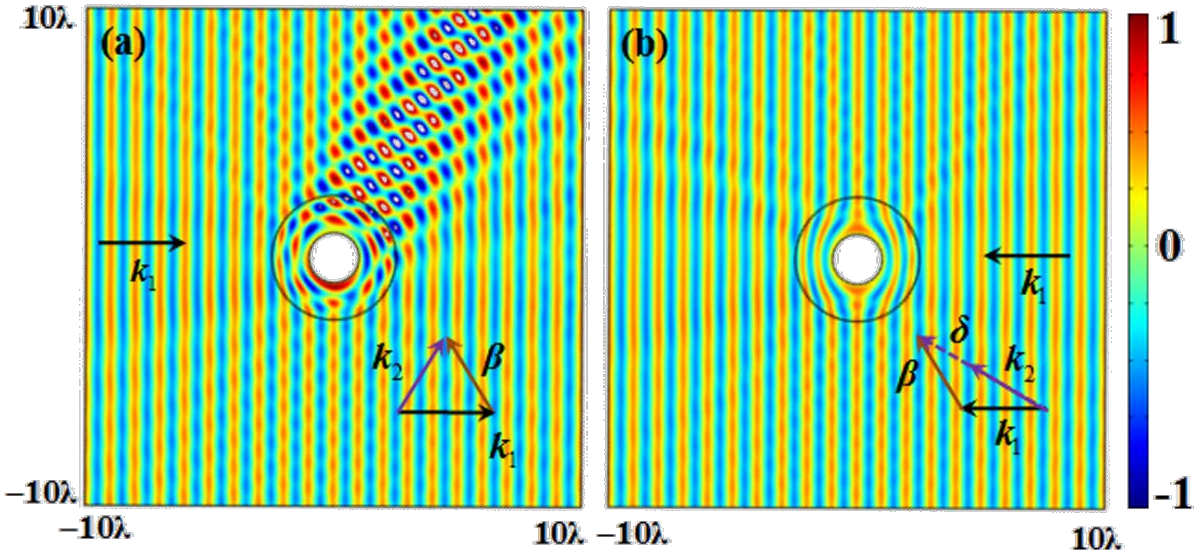
(reflecting)

For plane waves from the right

Phase mismatch: $\delta=\beta+k_1-k_2\neq 0$

(no reflecting)

Unidirectional light reflection (forward)



Note: $k_1 \neq k_2$ due to reciprocity

PT Symmetry in Acoustics

The analogy between acoustic equations and Maxwell equations

$$-i\omega\mu_r(-H_r) = -\frac{1}{r}\frac{\partial(-E_z)}{\partial\theta}, \qquad -i\omega\rho_\theta v_\theta = -\frac{1}{r}\frac{\partial p}{\partial\theta},$$

$$-i\omega\mu_\theta H_\theta = -\frac{\partial(-E_z)}{\partial r}, \quad \Longleftrightarrow \quad -i\omega\rho_r v_r = -\frac{\partial p}{\partial r},$$

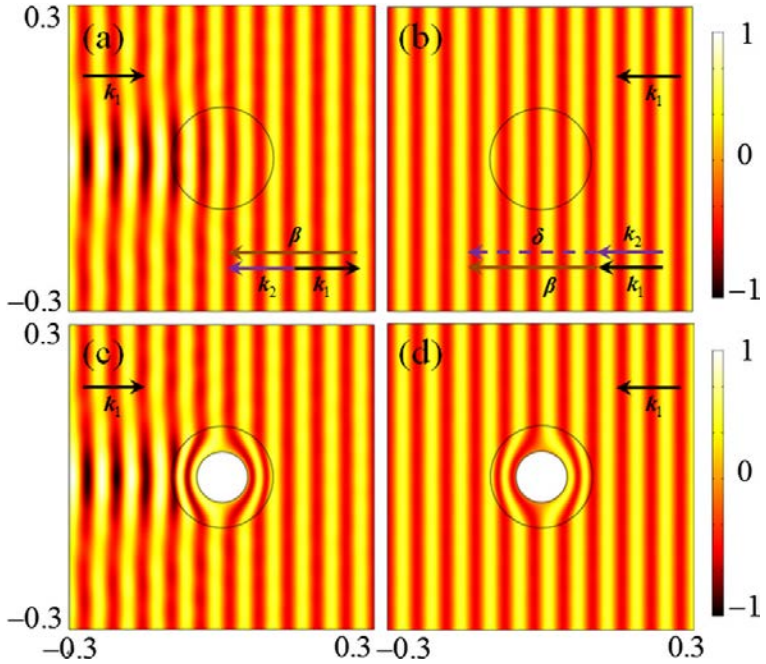
$$-i\omega\varepsilon_z(-E_z) = -\frac{1}{r}\frac{\partial(rH_\theta)}{\partial r} - \frac{1}{r}\frac{\partial(-H_r)}{\partial\theta} \qquad -i\omega\kappa^{-1}p = -\frac{1}{r}\frac{\partial(rv_r)}{\partial r} - \frac{1}{r}\frac{\partial v_\theta}{\partial\theta}$$

$$\begin{bmatrix} p & v_r & v_\theta & \rho_r & \rho_\theta & \kappa^{-1} \\ -E_z & H_\theta & -H_r & \mu_\theta & \mu_r & \varepsilon_z \end{bmatrix}$$

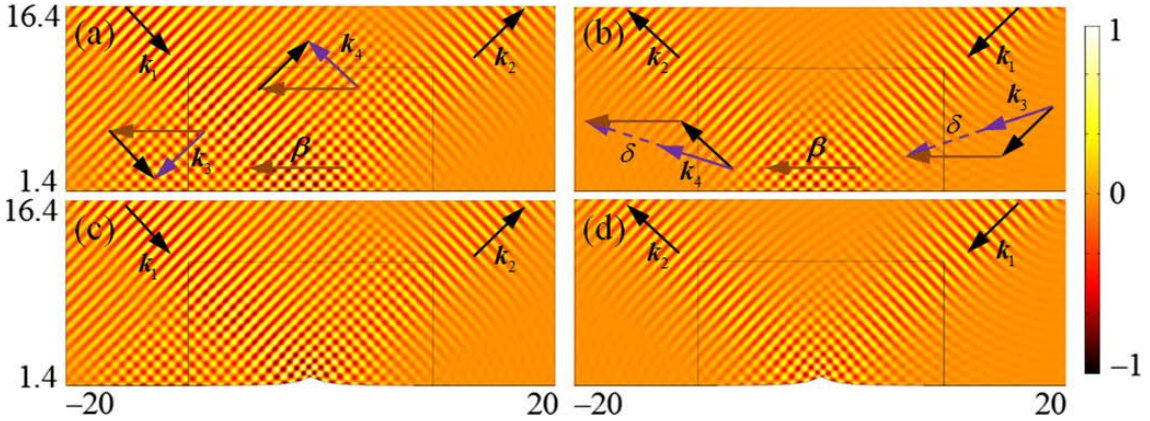
H. Chen and C. T. Chan, J. Phys. D: Appl. Phys. 43, 113001 (2010)

PT Symmetry in Acoustics

Unidirectional sound reflection



Unidirectional acoustic carpet cloaking



Exceptional Point:

$$\lambda_1 = \lambda_2 \text{ or } T = 1$$

$$S^*(k) = S^{-1}(k)$$

PT Symmetry

$$S(k) = \begin{pmatrix} t & r_R \\ r_L & t \end{pmatrix}$$

$$\lambda_{1,2} = t[1 \pm i\sqrt{(1-T)/T}]$$

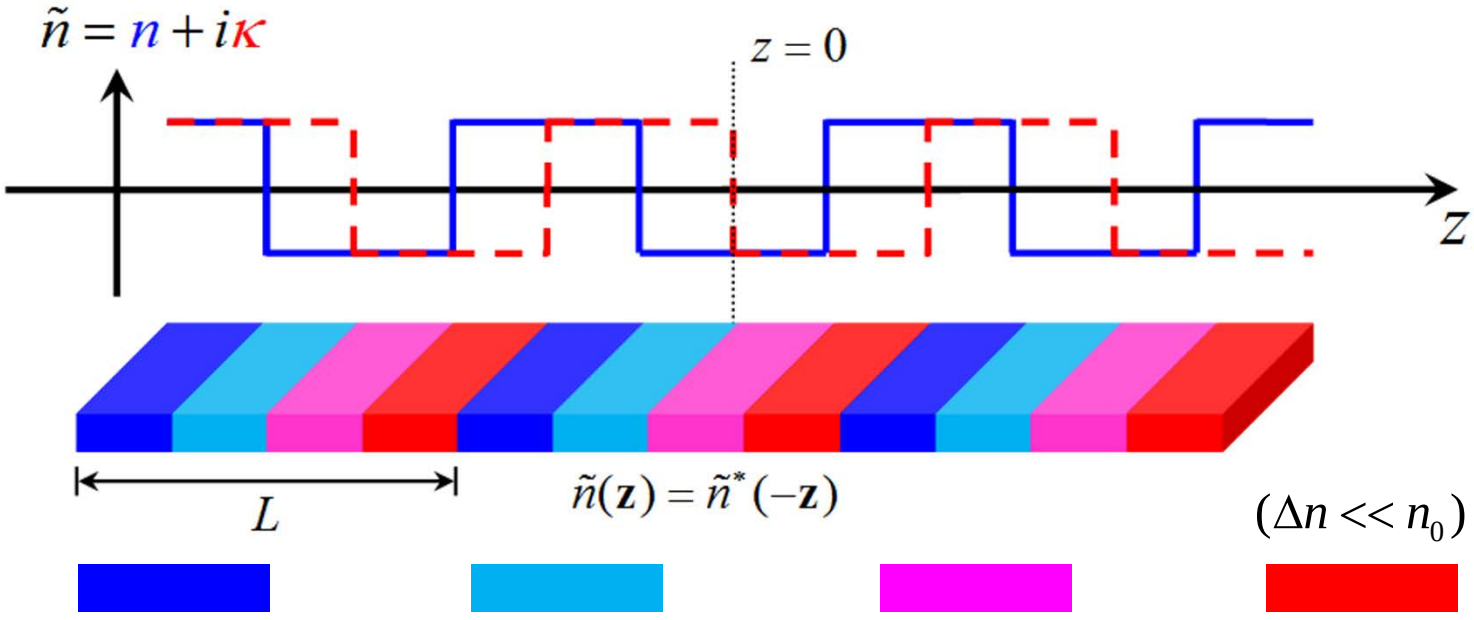
X. Zhu et al., Phys. Rev. X 4, 031042 (2014)

Other Unidirectional Phenomena in PT Materials

1. Bidirectional transparency and one-way light enhancement

2. Bifurcated Fabry-Pèrot resonances with unidirectional field enhancement

Bidirectional Transparency in PT Materials



At the Bragg's condition $\lambda = 2n_0 L$

$$r_L = \frac{(M_{11} + M_{12}n_0)n_0 - (M_{21} + M_{22}n_0)}{(M_{11} + M_{12}n_0)n_0 + (M_{21} + M_{22}n_0)}, \quad r_R = \frac{(M_{11} + M_{12}^* n_0)n_0 - (M_{21}^* + M_{22}n_0)}{(M_{11} + M_{12}^* n_0)n_0 + (M_{21}^* + M_{22}n_0)},$$

$$t = \frac{2n_0}{(M_{11} + M_{12}n_0)n_0 + (M_{21} + M_{22}n_0)}.$$

X. Zhu et al., Opt. Express, 22, 18401 (2014)

For the coefficients

$$M_{11} = m_{11}\frac{\sin\left(N(P-1)\pi/P\right)}{\pi/P} - \frac{\sin\left((N-1)(P-1)\pi/P\right)}{\pi/P}$$

$$M_{21} = m_{21}\frac{\sin\left(N(P-1)\pi/P\right)}{\pi/P} \quad M_{12} = m_{12}\frac{\sin\left(N(P-1)\pi/P\right)}{\pi/P}$$

$$M_{22} = m_{22}\frac{\sin\left(N(P-1)\pi/p\right)}{\pi/p} - \frac{\sin\left((N-1)(P-1)\pi/p\right)}{\pi/p}$$

Transfer matrix of one unit-cell

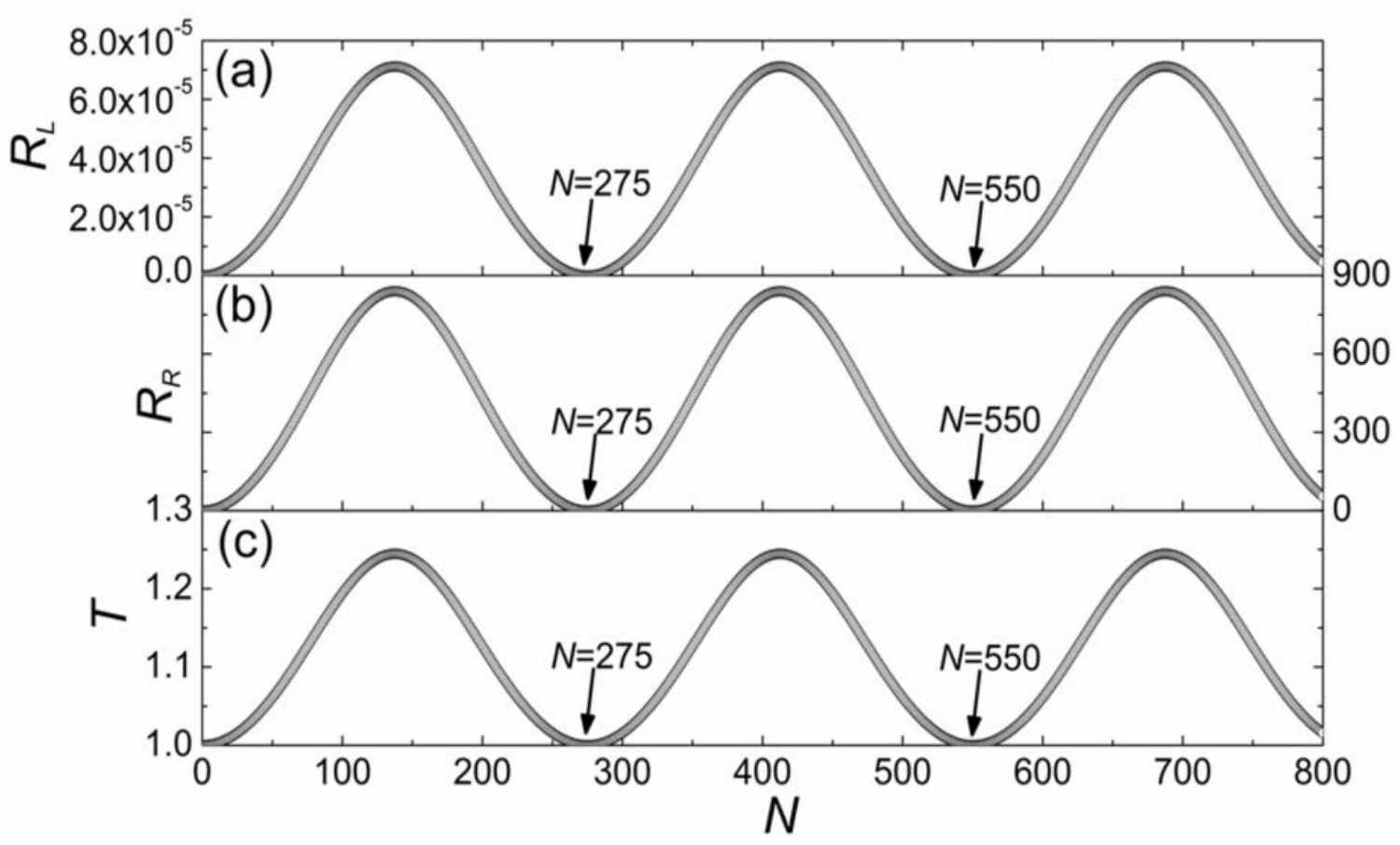
$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \prod_{j=1}^{4} \begin{bmatrix} \cos(k_0 n_j L_j) & -\frac{i}{n_j}\sin(k_0 n_j L_j) \\ -i n_j \sin(k_0 n_j L_j) & \cos(k_0 n_j L_j) \end{bmatrix}$$

The coefficient $P = \frac{11}{9}\left(\frac{n_0}{\Delta n}\right)^2$

When $N = mP$ We obtain $M_{11}, M_{22} = 1,\ M_{12}, M_{21} = 0 \rightarrow r_{L(R)} = 0,\ t = 1$

$m = 1, 2, 3, \cdots\cdots$ or $R_{L(R)} = 0,\ T = 1$

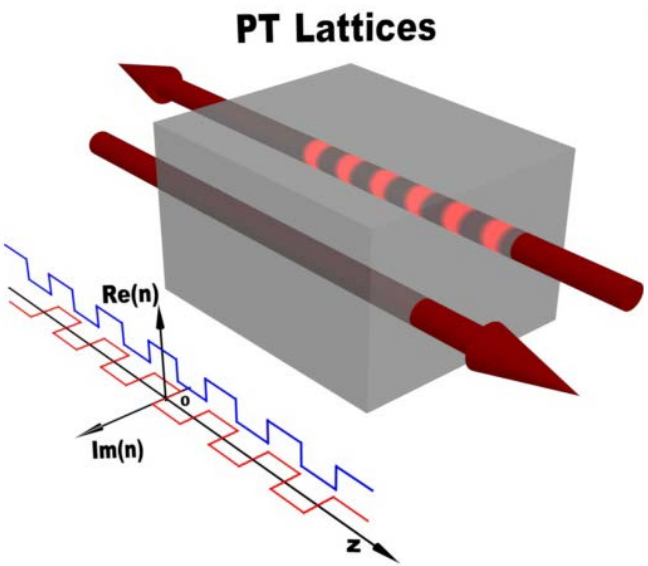
One Numerical Example

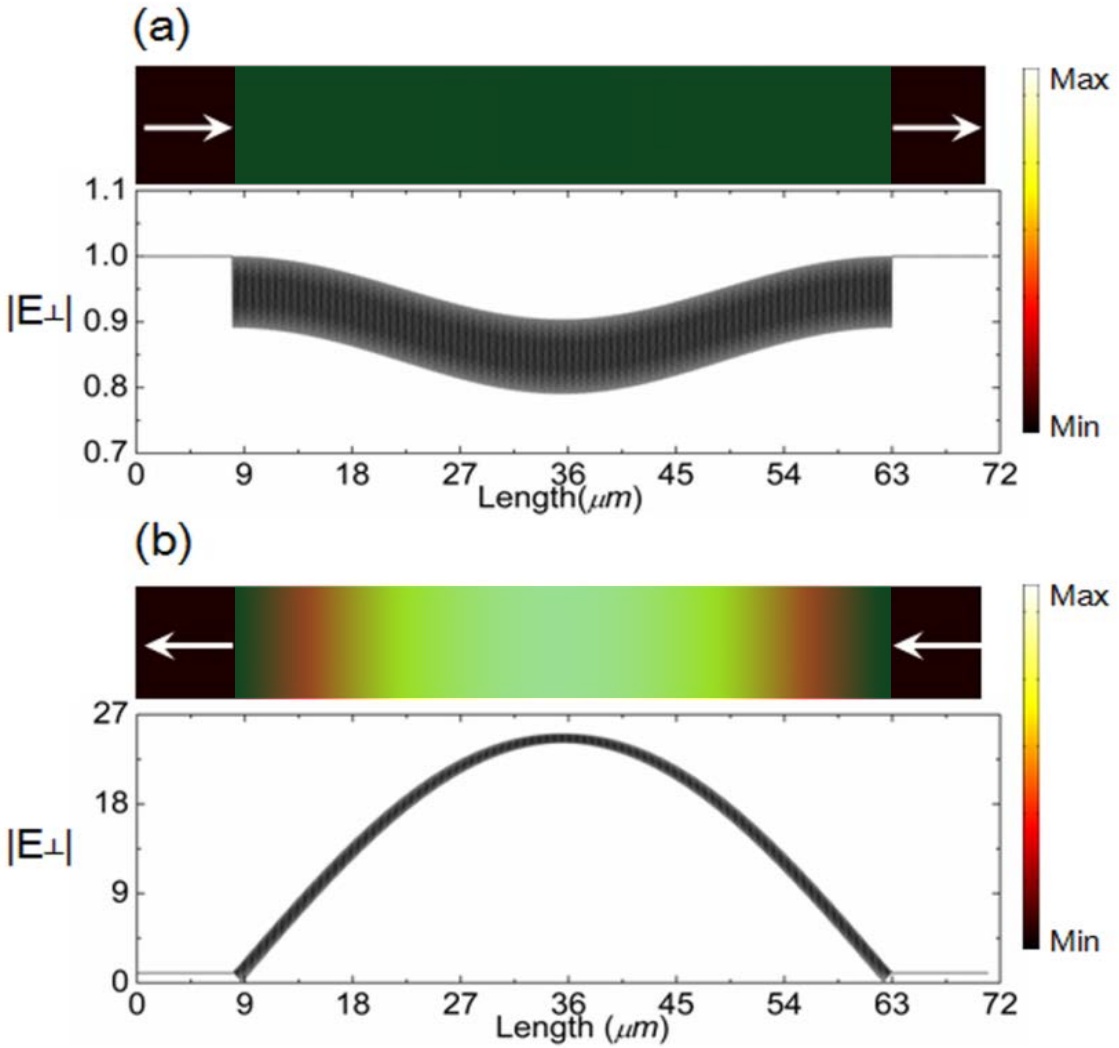


n_0=3, Δn=0.2, λ=1200 nm, $L=\lambda/(2n_0)$=200 nm

$$P = \frac{11}{9}\left(\frac{n_0}{\Delta n}\right)^2 = \frac{11}{9}\left(\frac{3}{0.2}\right)^2 = 275$$

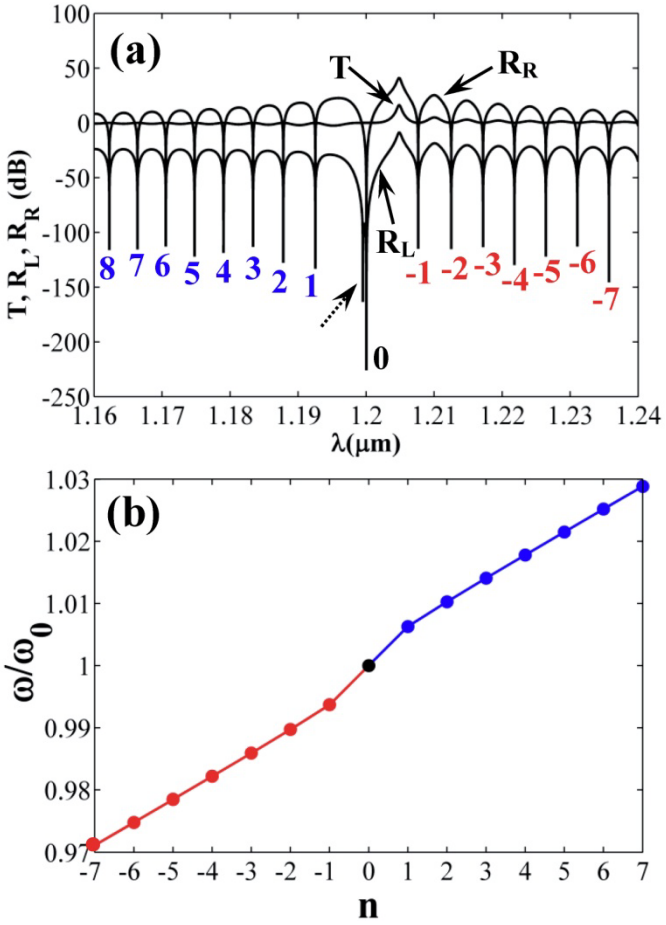
One-way Light Enhancement





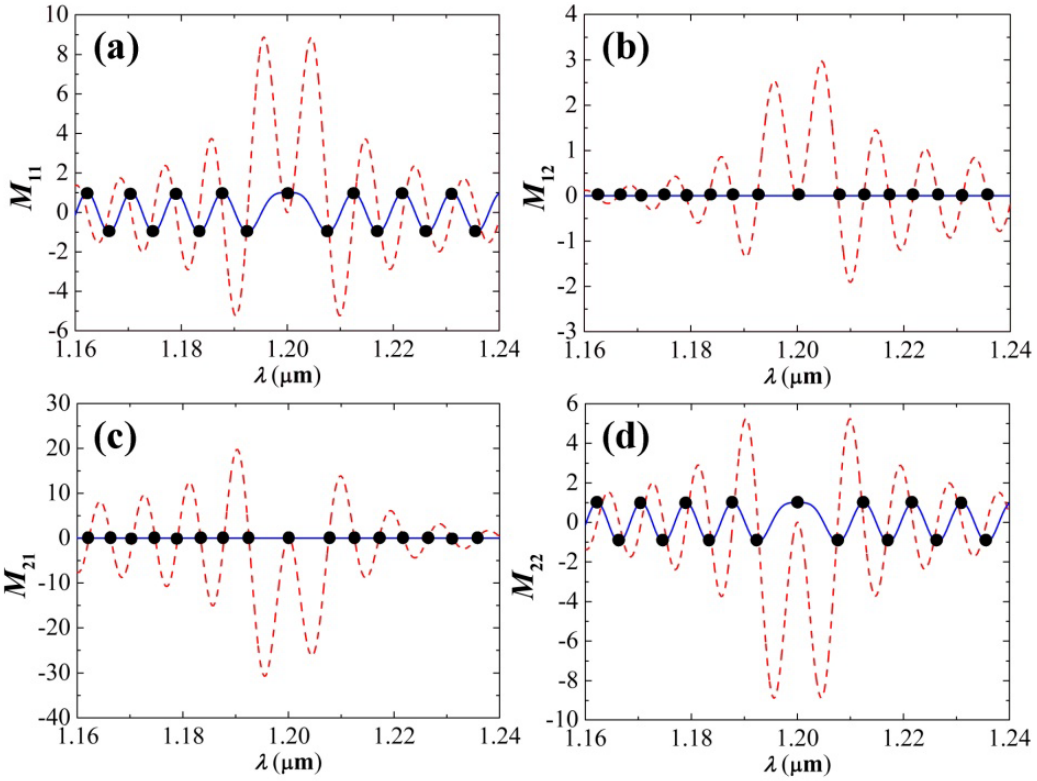
$N = 275$

Bifurcated Fabry-Pèrot Resonances



FP Resonances

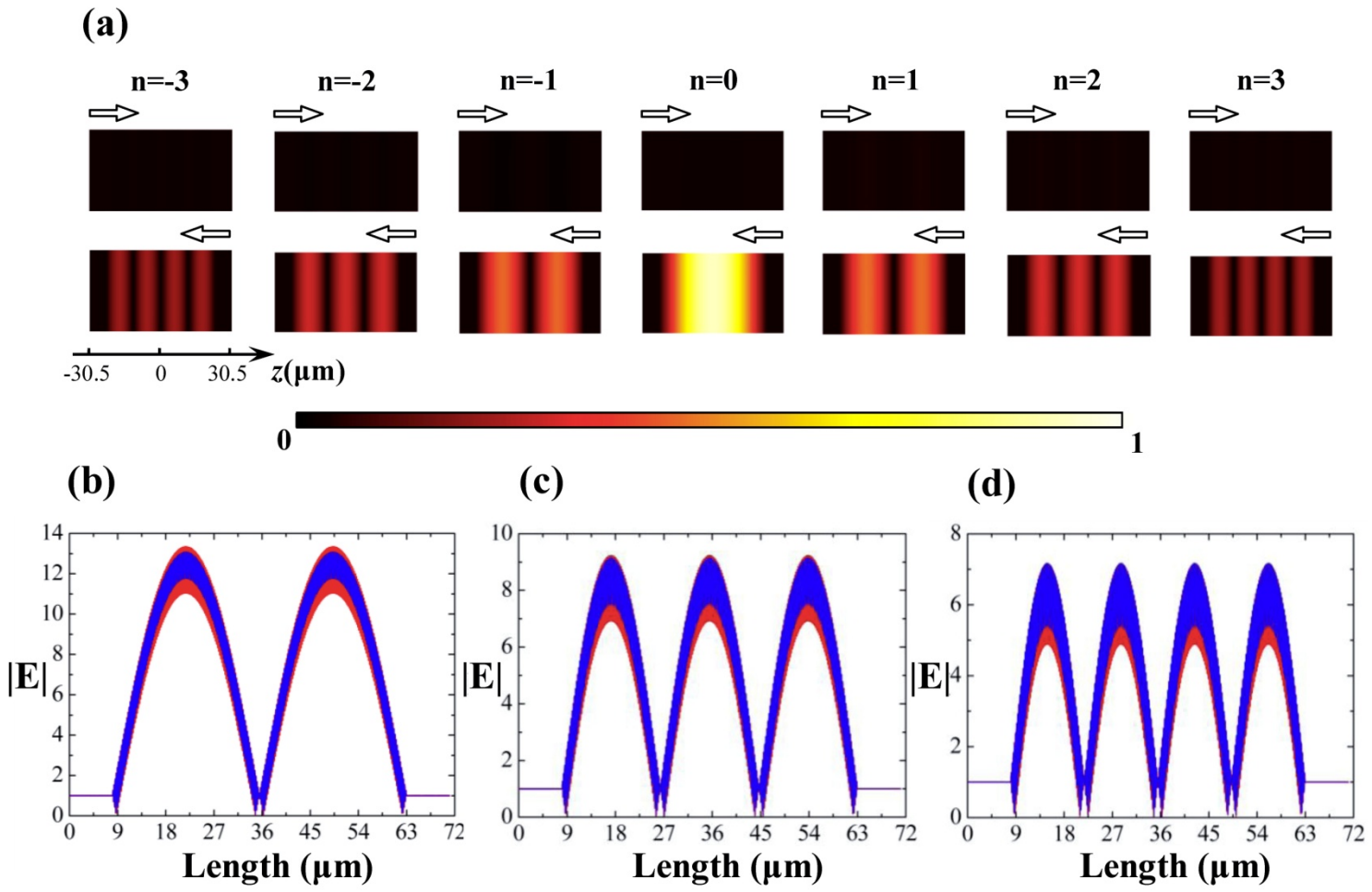
$$R_{L(R)} = 0,\ T = 1$$



$$M_{11} = M_{22} = 1,\ M_{12} = M_{21} = 0 \rightarrow r_{L(R)} = 0,\ t = 1$$

$$M_{11} = M_{22} = -1,\ M_{12} = M_{21} = 0 \rightarrow r_{L(R)} = 0,\ t = -1$$

One-way Light Enhancement



Summary

PT Optic/Acoustic System: $S(k)=\begin{pmatrix} t & r_R \\ r_L & t \end{pmatrix}$

$$|T-1|=\sqrt{R_L R_R} \xrightarrow{T=1} \begin{cases} R_L=0,\ R_R\neq 0 \\ R_L=0,\ R_R=0 \\ R_L=0,\ R_R=\infty \end{cases}$$

Z. Lin et al., Phys. Rev. Lett. 106, 213901 (2011)
L. Feng et al., Nat. Mater. 12, 108(2013)……
X. Zhu et al., Opt. Lett., 38, 15, 2821(2013)
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X. Zhu et al., Opt. Express, 22, 18401 (2014)

H. Ramezani et al., Phys. Rev. Lett. 113, 263905 (2014)