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IAS WINTER SCHOOL & WORKSHOP
Advanced Concepts in Wave Physics:
Topology and Parity-time Symmetries
11-15 Jan 2016

Optical force on toroidal nanostructures: toroidal dipole versus renormalized electric dipole

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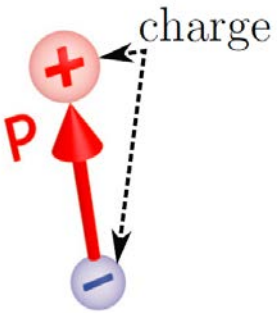
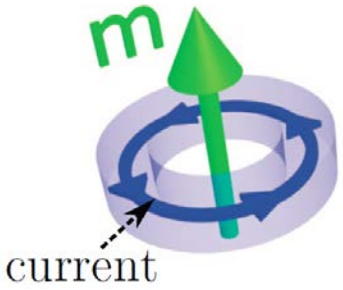
Collaborators: Dr. Shubo Wang, Prof. Zhifang Lin, Prof. C. T. Chan

Background

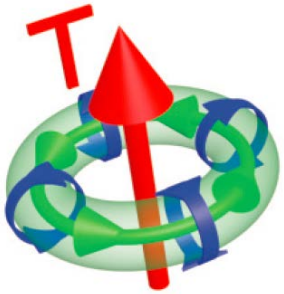
Multipoles



Electric dipole

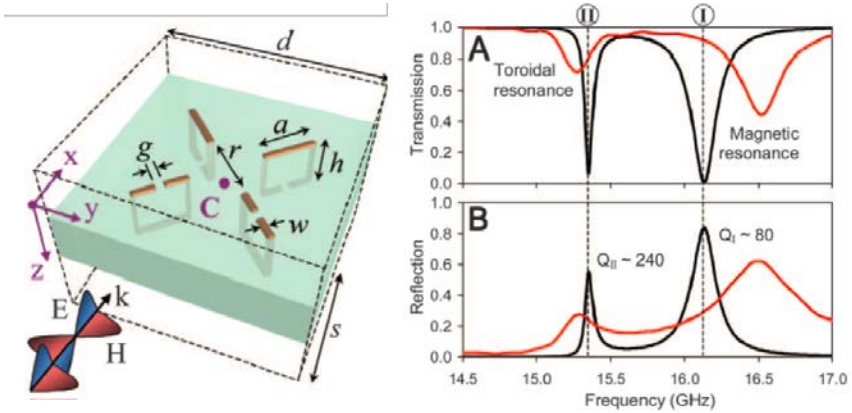


Magnetic dipole



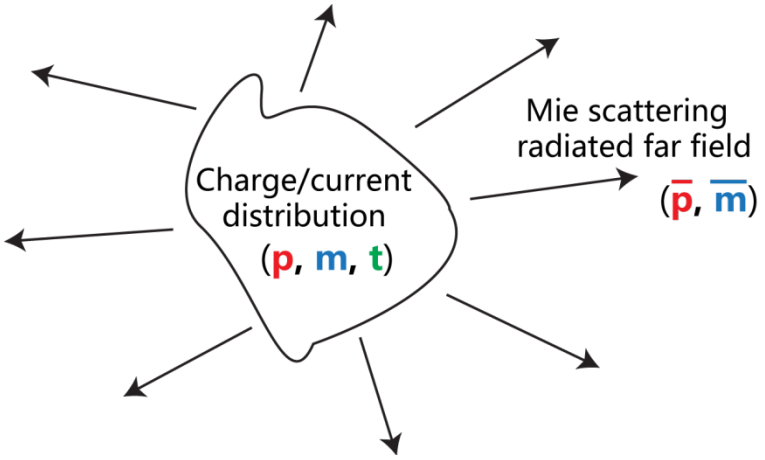
Toroidal dipole

Experimental demonstration of toroidal dipole



T. Kaelberer, et al., Science **330**, 1510 (2010)

Confusion about the toroidal multipoles



Background

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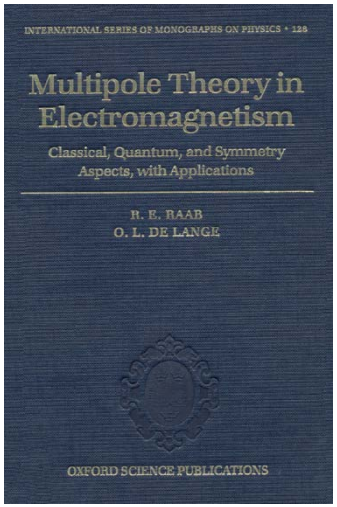
Classical Electrodynamics
Third Edition

John David Jackson
Professor Emeritus of Physics,
University of California, Berkeley

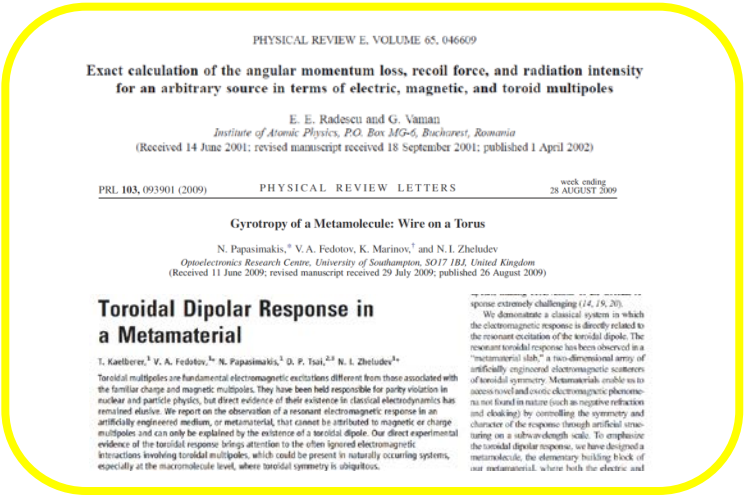
Absorption and Scattering of Light by Small Particles

CRAIG F. BOHREN
Distinguished Professor of Meteorology
The Pennsylvania State University

DONALD R. HUFFMAN
Regents Professor of Physics
The University of Arizona



Classical textbooks



PHYSICAL REVIEW E, VOLUME 65, 046609

Exact calculation of the angular momentum loss, recoil force, and radiation intensity for an arbitrary source in terms of electric, magnetic, and toroid multipoles

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Institute of Atomic Physics, P.O. Box MG-6, Bucharest, Romania
(Received 14 June 2001; revised manuscript received 18 September 2001; published 1 April 2002)

PRL 103, 093901 (2009) PHYSICAL REVIEW LETTERS week ending 28 AUGUST 2009

Gyrotropy of a Metamolecule: Wire on a Torus

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(Received 11 June 2009; revised manuscript received 29 July 2009; published 26 August 2009)

Toroidal Dipolar Response in a Metamaterial

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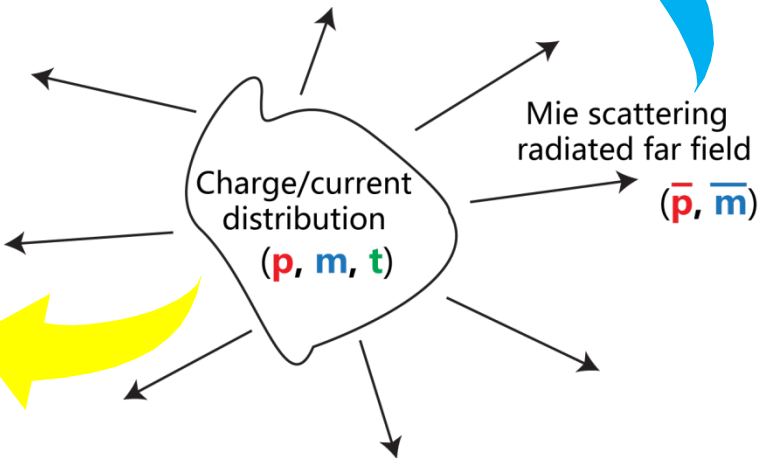
Toroidal multipoles are fundamental electromagnetic excitations different from those associated with the familiar charge and magnetic multipoles. They have been held responsible for parity violation in nuclear and particle physics, but direct evidence of their existence in classical electrodynamics has remained elusive. We report on the observation of a resonant electromagnetic response in an artificially engineered medium, or metamaterial, that cannot be attributed to magnetic or charge multipoles and can only be explained by the existence of a toroidal dipole. Our direct experimental evidence of the toroidal response brings attention to the often ignored electromagnetic interactions involving toroidal multipoles, which could be present in naturally occurring systems, especially at the macromolecule level, where toroidal symmetry is ubiquitous.



Multipole definition

Source-representation

primitive multipoles

$$p_i = \int_V r_i' \rho dv', \quad m_i = \frac{1}{2}\int_V (\mathbf{r}' \times \mathbf{J})_i \, dv',$$

$$q_{ij}^{(e)} = \int_V r_i' r_j' \rho dv', \quad q_{ij}^{(m)} = \frac{2}{3}\int_V (\mathbf{r}' \times \mathbf{J})_i \, r_j' dv',$$

=

irreducible multipoles

$$p_i = \int_V r_i' \rho dv', \quad m_i = \frac{1}{2}\int_V (\mathbf{r}' \times \mathbf{J})_i \, dv',$$

$$Q_{ij}^{(e)} = \int_V \left(r_i' r_j' - \frac{1}{3}\mathbf{r}'^2 \delta_{ij} \right) \rho dv',$$

$$Q_{ij}^{(m)} = \frac{1}{3}\int_V \left[(\mathbf{r}' \times \mathbf{J})_i r_j' + (\mathbf{r}' \times \mathbf{J})_j r_i' \right] dv',$$

+

toroidal multipoles

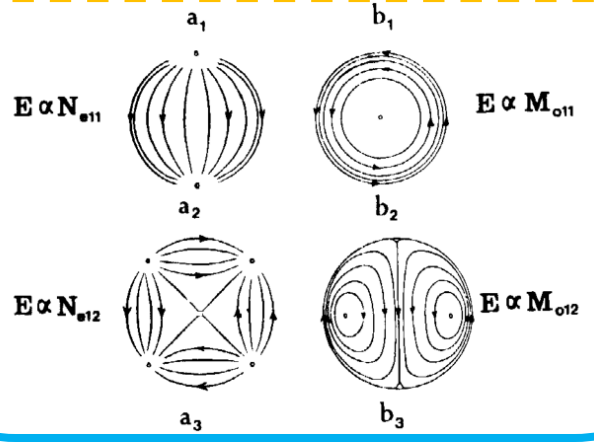
$$t_i = \frac{1}{10}\int_V \left[(\mathbf{r}' \cdot \mathbf{J})\mathbf{r}' - 2\mathbf{r}'^2 \mathbf{J} \right]_i dv',$$

Field-representation

renormalized multipoles

$$\mathbf{E}_s = \sum_{n,m} iE_{mn} \left[a_{mn} \mathbf{N}_{mn}^{(3)}(k,\mathbf{r}) + b_{mn} \mathbf{M}_{mn}^{(3)}(k,\mathbf{r}) \right],$$

$$\mathbf{H}_s = \frac{k}{\omega\mu_0} \sum_{n,m} E_{mn} \left[b_{mn} \mathbf{N}_{mn}^{(3)}(k,\mathbf{r}) + a_{mn} \mathbf{M}_{mn}^{(3)}(k,\mathbf{r}) \right].$$



irreducible dipoles: p, m

toroidal dipole: t

renormalized dipoles: $\overline{\mathbf{p}}$, $\overline{\mathbf{m}}$

Two viewpoints of optical forces

Optical force

Source-representation

Taylor expansion of vector potential

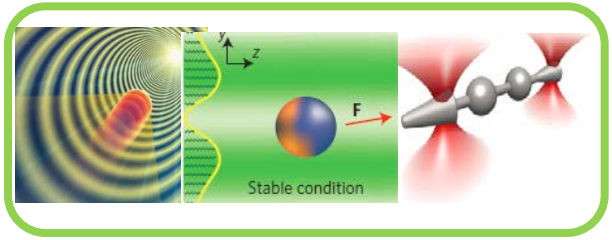
$$\mathbf{A}(\mathbf{r}) = \mu_0 / (4\pi) \int_V \left[\mathbf{J}(\mathbf{r}') e^{i\omega R/c} / R \right] dv'$$

Lorentz force

$$\mathbf{F}^{\mathbf{p}} = \frac{1}{2} \mathrm{Re}\left[\left(\nabla \mathbf{E}_i^* \right) \cdot \mathbf{p} \right] \quad \mathbf{F}^{\mathbf{m}} = \frac{1}{2} \mathrm{Re}\left[\left(\nabla \mathbf{B}_i^* \right) \cdot \mathbf{m} \right]$$

$$\mathbf{F}^{\mathbf{t}} = -\frac{k}{2c} \mathrm{Im}\left[\left(\nabla \mathbf{E}_i^* \right) \cdot \mathbf{t} \right]$$

$$\mathbf{F}^{\mathrm{int}} = -\frac{k^4}{12\pi\varepsilon_0 c} \mathrm{Re}\left[\mathbf{p} \times \mathbf{m}^* \right] + \frac{k^5}{12\pi\varepsilon_0 c^2} \mathrm{Im}\left[\mathbf{m} \times \mathbf{t}^* \right]$$



Field-representation

Based on vector spherical functions

$$\mathbf{E}_s = \sum_{n,m} iE_{mn} \left[a_{mn} \mathbf{N}_{mn}^{(3)}(k, \mathbf{r}) + b_{mn} \mathbf{M}_{mn}^{(3)}(k, \mathbf{r}) \right]$$

Variation of Maxwell stress tensor

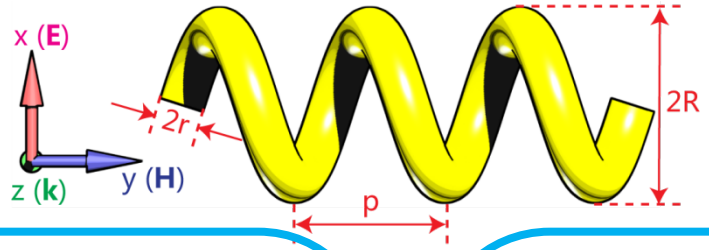
$$F_z^{\bar{\mathbf{p}}} = \frac{2\pi\varepsilon_0}{k^2} |E_0|^2 \sum_{n=1,m} \mathrm{Re}\left[g_{mn} a_{m,n} p_{m,n+1}^* + l_{mn} q_{m,n} a_{m,n}^* \right]$$

$$F_z^{\bar{\mathbf{m}}} = \frac{2\pi\varepsilon_0}{k^2} |E_0|^2 \sum_{n=1,m} \mathrm{Re}\left[g_{mn} b_{m,n} q_{m,n+1}^* + l_{mn} p_{m,n} b_{m,n}^* \right]$$

$$F_z^{\overline{\mathrm{int}}} = -\frac{4\pi\varepsilon_0}{k^2} |E_0|^2 \sum_{n=1,m} \mathrm{Re}\left(l_{mn} a_{m,n} b_{m,n}^* \right)$$

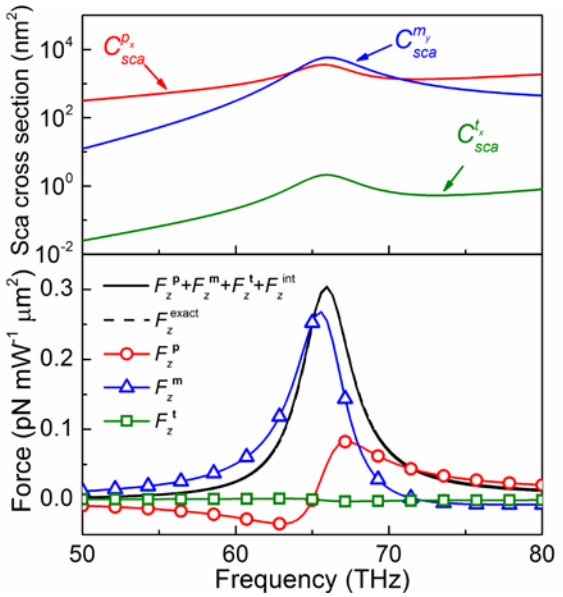
Optical force on single helix

Magnetic dipole

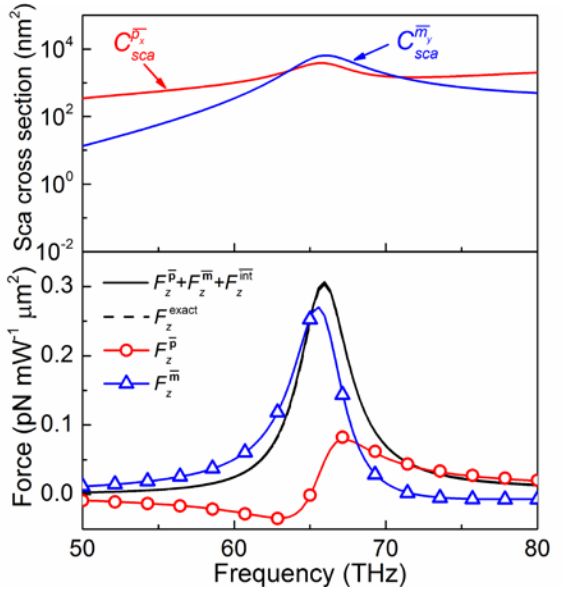


r = 30 nm
R = 100 nm
p = 200 nm

Source-representation



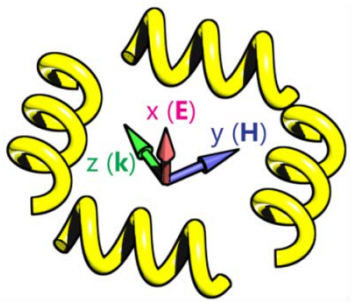
Field-representation

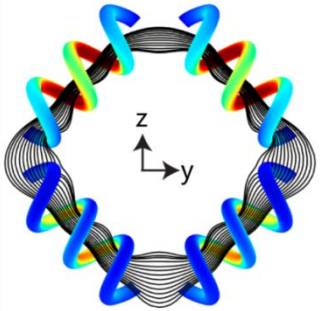


The effect from the toroidal dipole is completely masked by the irreducible electric and magnetic dipoles, obscuring any distinction between the irreducible and the renormalized dipoles, as in most conventional structures.

Optical force on toroidal nanostructures

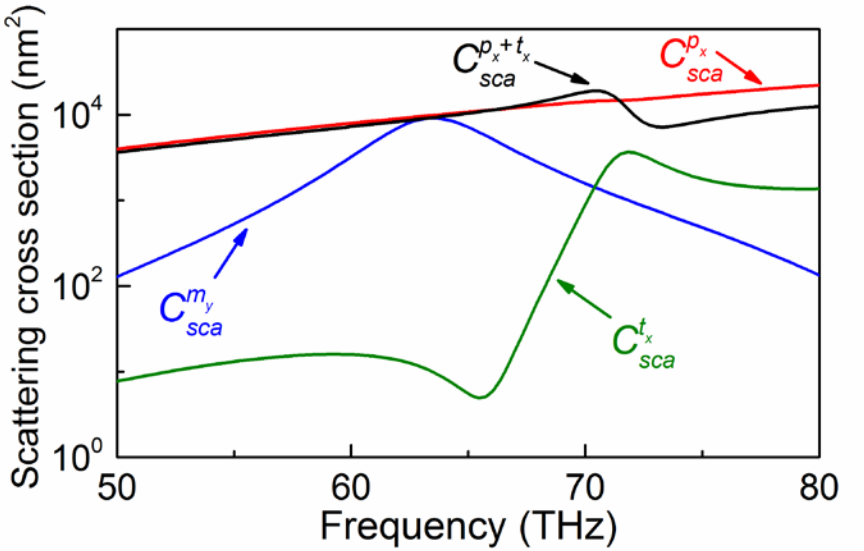
Source-representation

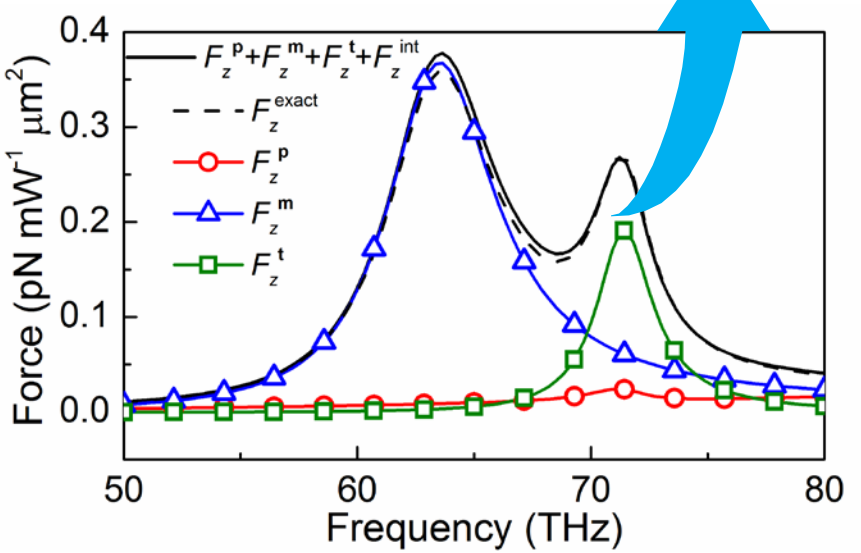




How to explain the resonant force in the field-representation ?

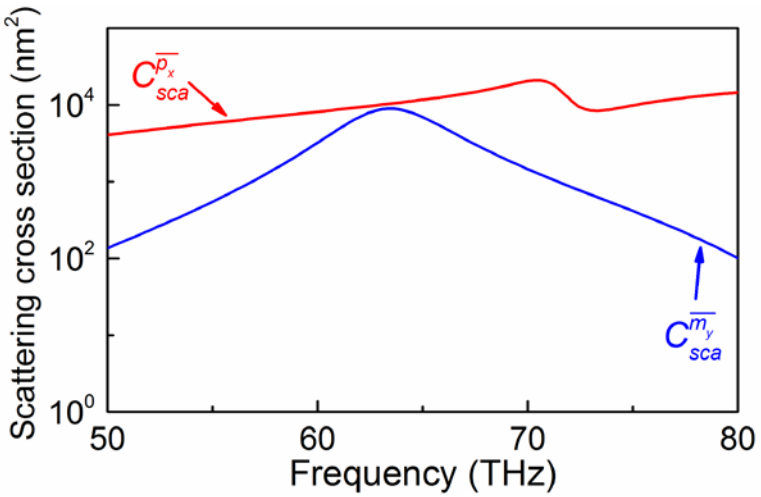
The toroidal dipole is necessary for the description of optical forces in the source-representation.



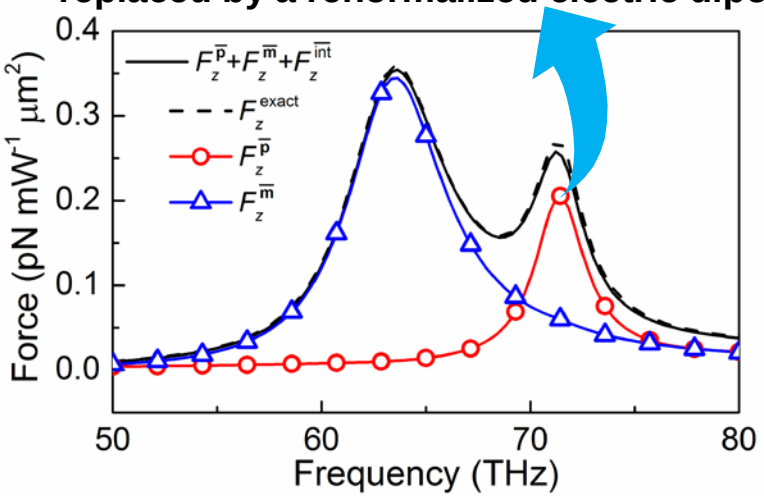


Optical force on toroidal nanostructures

Field-representation



The force contribution of toroidal dipole is replaced by a renormalized electric dipole.



Relationship of the multipoles between source-representation and field-representation

$$\mathbf{F}^{\mathbf{p}}+\mathbf{F}^{\mathbf{t}}\approx\mathbf{F}^{\bar{\mathbf{p}}},\quad \mathbf{F}^{\mathbf{m}}\approx\mathbf{F}^{\bar{\mathbf{m}}}$$

$$C^{\mathbf{p}}_{\mathrm{sca}}+C^{\mathbf{t}}_{\mathrm{sca}}\approx C^{\bar{\mathbf{p}}}_{\mathrm{sca}},\quad C^{\mathbf{m}}_{\mathrm{sca}}\approx C^{\bar{\mathbf{m}}}_{\mathrm{sca}}$$

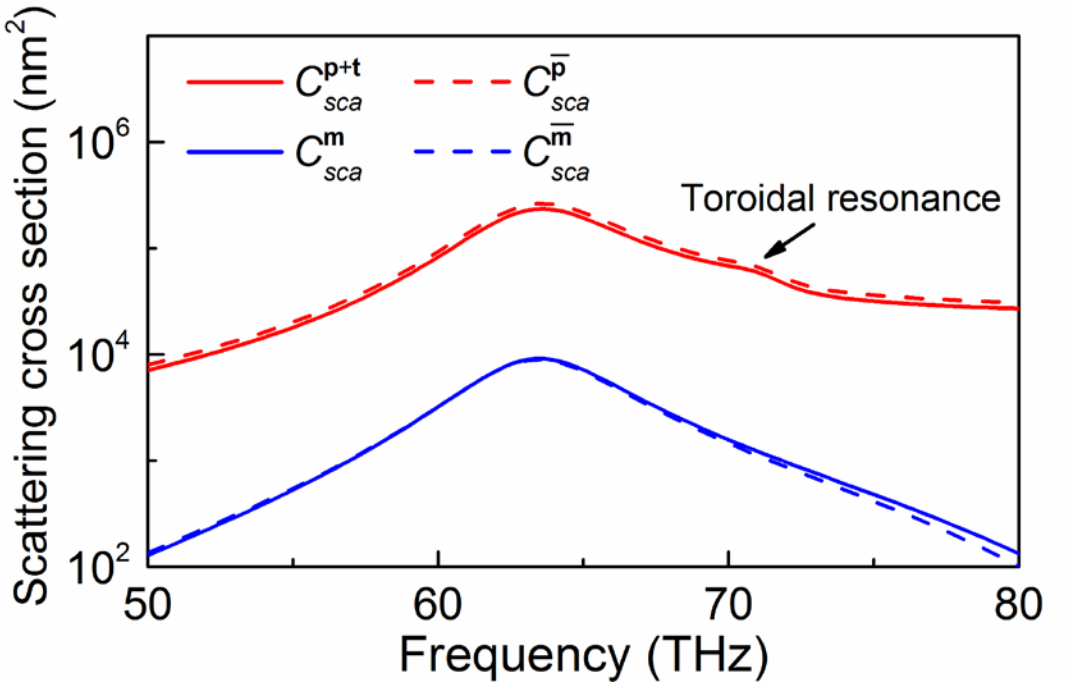
$$\mathbf{p}+\frac{ik}{c}\mathbf{t}\approx\bar{\mathbf{p}},\quad \mathbf{m}\approx\bar{\mathbf{m}}$$

Electric radiation field: (source representation)

$$\vec{E}=\frac{\kappa^2}{4\pi\varepsilon_0}\frac{e^{i\kappa r}}{r}\left\{\vec{n}\times(\vec{p}\times\vec{n})+\frac{1}{c}(\vec{m}\times\vec{n})+\frac{ik}{2}\vec{n}\times\left[\vec{n}\times(\overleftrightarrow{Q}_e\cdot\vec{n})\right]\right.$$
$$\left.+\frac{ik}{2c}\vec{n}\times(\overleftrightarrow{Q}_m\cdot\vec{n})+\frac{k^2}{6}\vec{n}\times\left[\vec{n}\times(\hat{\Omega}_e\cdot\vec{n}\cdot\vec{n})\right]+\frac{ik}{c}\vec{n}\times(\vec{t}\times\vec{n})\right\}$$

Detection of toroidal resonance

Detection of the toroidal resonance



Source-representation

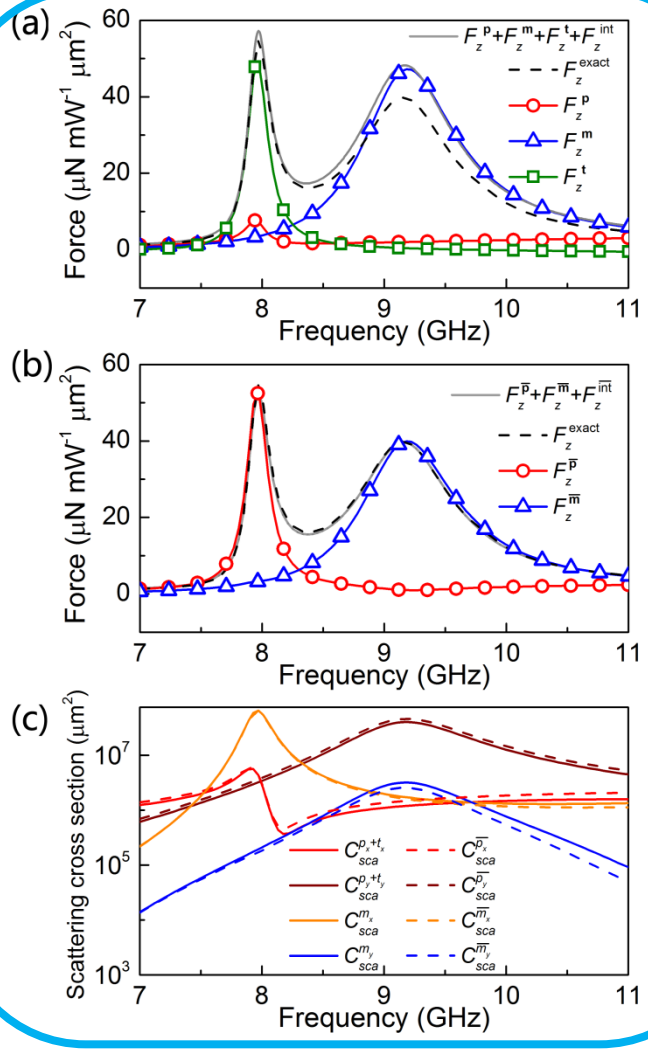
$$\begin{cases} C_{sca}^{\mathbf{p}} = \dfrac{Z_0\omega^4}{3\pi\varepsilon_0 c^3 |E_0|^2}|\mathbf{p}|^2 \\ C_{sca}^{\mathbf{p+t}} = \dfrac{Z_0\omega^4}{3\pi\varepsilon_0 c^3 |E_0|^2}\left|\mathbf{p}+\dfrac{ik}{c}\mathbf{t}\right|^2 \end{cases}$$

Field-representation

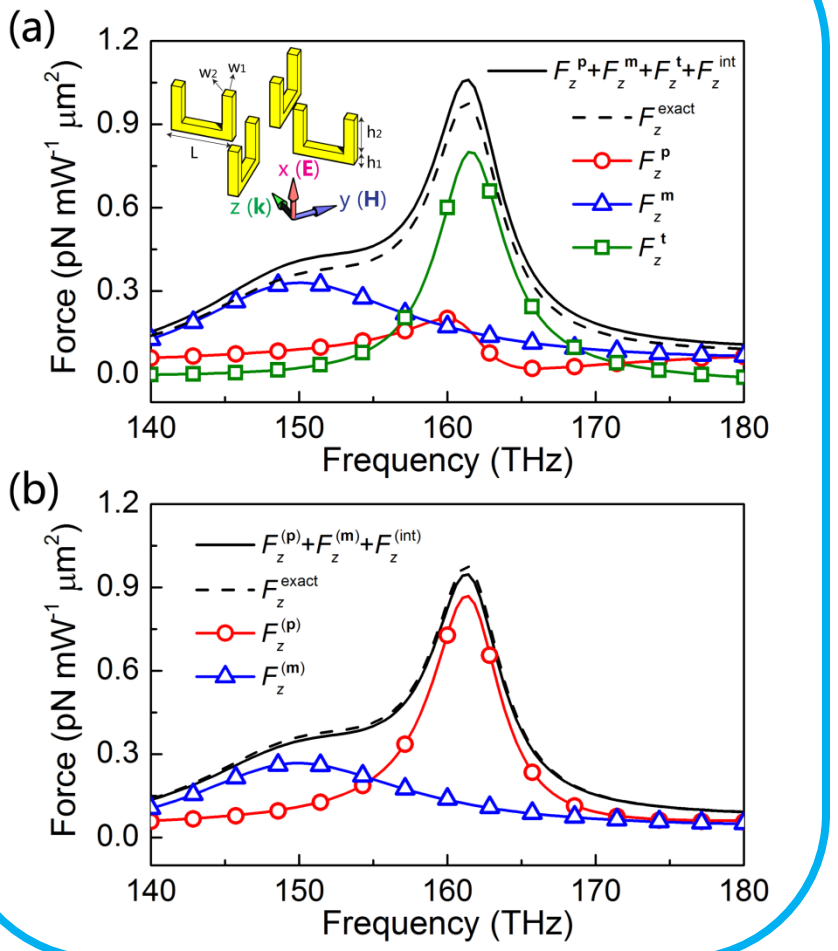
$$\begin{cases} C_{sca}^{\bar{\mathbf{p}}} = \dfrac{6\pi}{k^2}\left(|A_{e11}|^2+|A_{o11}|^2+|A_{e01}|^2\right) \\ C_{sca}^{\bar{\mathbf{m}}} = \dfrac{6\pi}{k^2}\left(|B_{e11}|^2+|B_{o11}|^2+|B_{e01}|^2\right) \end{cases}$$

Scattering cross section	The total number of photons scattered
Optical force	The number of photons scattered as well as their distributions

Some other demonstrations



SRR structures



Summary

Some confusions in the understanding of various multipoles have been clarified by introducing and distinguishing the primitive multipoles, irreducible multipoles, and renormalized multipoles. The toroidal dipole is meaningful and necessary in the source representation while its contribution can be completely incorporated into the renormalized electric dipole in the far field.

We also show that optical force enables the observation of the toroidal response of a nanostructure even when its effect on scattering power is overwhelmed by the conventional multipoles.

Phys. Rev. A 92, 043804 (2015)

Thanks for your attention!