Optimal Approximation with Sparsely Connected Deep Neural Networks

Gitta Kutyniok

Institute of Mathematics, Technische Universität Berlin, Germany

Email: kutyniok (at) math.tu-berlin.de

Despite the outstanding success of deep neural networks in real-world applications, most of the related research is empirically driven and a mathematical foundation is almost completely missing. One central task of a neural network is to approximate a function, which for instance encodes a classification task. In this talk, we will be concerned with the question, how well a function can be approximated by a neural network with sparse connectivity. Using methods from approximation theory and applied harmonic analysis, we will derive a fundamental lower bound on the sparsity of a neural network. By explicitly constructing neural networks based on certain representation systems, so-called α-shearlets, we will then demonstrate that this lower bound can in fact be attained. Finally, we present numerical experiments, which surprisingly show that already the standard backpropagation algorithm generates deep neural networks obeying those optimal approximation rates. This is joint work with H. B\"olcskei (ETH Zurich), P. Grohs (Uni Vienna), and P. Petersen (TU Berlin).